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The ϕ^4 -and Penner models of 2d Quantum gravity, the moduli space of curves and properties of 2-cell embeddings of graphs in Riemann surfaces

A *map* is a 2-cell embedding of a graph in a Riemann surface. The generating series for a class of maps is called the *map series* for the class. I shall discuss two questions, one from mathematical physics and the other from algebraic geometry, where map theory reveals the presence of deeper structure and connexions between the two.

I) The ϕ^4 -model and $\log(1-\phi)^{-1}$ -model (due to Penner) are early models of topological quantum field theory. The relationship between the partition functions for these two models may be explained as a consequence of a functional relationship between two classes of maps in orientable surfaces, one in which all vertices have degree 4 and the other in which there is no restriction on vertex degrees. Moreover, comparable relationships hold for other classes of maps and there is evidence of a natural bijection accounting for these relationships.

II) The generating series for the virtual Euler characteristics for the moduli spaces of complex and for real algebraic curves, respectively, may be shown to be specialisations of the map series for all surfaces through an algebraic parameter associated with Jack symmetric functions. This parameter is conjectured to have an interpretation as an invariant of maps, which then opens the possibility of passing it through the Strebel derivative construction used by Harer and Zagier, to the level of the moduli spaces.

In this talk I shall show how these conjectures arose in the first place.