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Cluster algebras arising from preinjective modules

Let Q be a quiver without oriented cycles, \mathfrak{n} the positive part of the symmetric Kac–Moody algebra of type Q and $\Lambda = \mathbb{C}\overline{Q}/\langle \sum_{a \in Q} [a, \bar{a}] \rangle$ the corresponding preprojective algebra. Let $M = \bigoplus_{i=1}^r M_i$ be a terminal representation of Q , i.e., the summands of M are a family of indecomposable preinjective representations, closed under successors. Then the full subcategory

$$\mathcal{C}_M = \{X \in \Lambda\text{-mod}_0 \text{ with } X|_Q \in \text{Add}(M)\}$$

is a Frobenius category and its stable category is 2-Calabi–Yau. Moreover it has a canonical cluster tilting object. We can describe \mathcal{C}_M conveniently as the Δ -good modules of a quasi-hereditary algebra, in this setting, mutation can be described via Δ -dimension vectors.

We have a cluster character ϕ from $\Lambda\text{-mod}_0$ to the commutative ring $U(\mathfrak{n})^*$ which is compatible with our subcategories, so we get a cluster algebra structure on the “image” of \mathcal{C}_M . These cluster algebras are closely related to the coordinate rings of certain reduced double Bruhat cells.

This is a report on joint work with B. Leclerc and J. Schröer.