
Algebraic Stacks
Champs algébriques
(Org: **Ajneet Dhillon** (Western))

JAROD ALPER, Stanford University
Good moduli spaces for Artin stacks

I will introduce a stack-theoretic approach to geometric invariant theory and develop an intrinsic theory for associating to arbitrary Artin stacks schemes or algebraic spaces with nice geometric properties. I will define the notion of a good moduli space which simultaneously generalizes the existing notions of good GIT quotients and tame stacks. This theory will be used to explore the local structure of Artin stacks and in particular address the question concerning whether Artin stacks are locally quotient stacks.

AREND BAYER, University of Utah
Quantum cohomology of C^N/μ_r

We give an explicit construction of the moduli space $\overline{M}_{0,n}(B\mu_r)$ of genus-zero twisted stable maps to $B\mu_r$ (the classifying stack of a cyclic group) by a sequence of r -th root constructions on $\overline{M}_{0,n}$. By using a notion of weighted stable maps, we prove a formula for the total Chern class of the μ_r -eigenspaces of the Hodge bundle. This gives a formula for the Chern class of the obstruction bundle computing the Gromov–Witten invariants of $[C^N/\mu_r]$ for any linear group action of μ_r on C^N .

We deduce linear recursions for all genus-zero Gromov–Witten invariants of $[C^N/\mu_r]$.

This is joint work with Charles Cadman, arXiv:0705.2158.

RENZO CAVALIERI, University of Michigan, 530 Church Street, Ann Arbor, MI, USA
G-Hodge Integrals, Gerby Localization and the GW Theory of $[C^3/\mathbb{Z}_3]$

In this talk we discuss a successful approach to the computation of orbifold Gromov–Witten invariants, focusing in particular on the orbifold $[C^3/\mathbb{Z}_3]$, which has recently been source of interest to both physicists and mathematicians. Such invariants are interpreted in terms of G-Hodge integrals, and relations among G-Hodge integrals are obtained via Atiyah–Bott localization. In order to find sufficiently many relations to reconstruct all invariants, we turn our attention to moduli spaces of maps to gerbes over \mathbb{P}^1 .

This is joint work with C. Cadman.

MIKE FRIED, UC Irvine
Atomic Orbital-type cusps on Alternating Group Modular Towers

Reduced Hurwitz spaces—spaces of r -branched Riemann surface covers of the projective line—are dimension $r - 3$ moduli spaces. These stacks have cusps on their boundaries. They can have fine moduli, but often do not. In the form of **M(odular) T(owers)** they support conjectures generalizing modular curve statements. Other researchers use these to connect the *Inverse Galois Problem* and the *Strong Torsion Conjecture* (on abelian varieties).

Like Shimura varieties—some are special cases—each **MT** comes with a prime p . As many **MTs** attach to p as there are p -perfect finite groups. We get a hold on these spaces using a *sh-incidence pairing* on their cusps.

We will concentrate on applying the sh-incidence pairing to infinitely many **MTs** where the Main Conjectures are proved. We chose examples of Liu and Osserman, who proved a first connectedness result. Here the projective line covers have alternating groups as monodromy groups, $p = 2$ and $r = 4$ (so tower levels are upper half plane quotients, but not modular curves). I swear, the group theory is surprisingly easy.

We use a “Fried–Serre” spin-lifting formula to locate 2 cusps. Our computations were guided by the look of an *atomic orbital* in sh-incidence rows. By catching 2 cusps at tower level 1—though there are none at level 0—we prove the Main Conjecture. We end by comparing p cusps in **MTs** with those of modular curve towers.

YUFENG JIANG, University of Utah, 155S 1400E JWB 233, Salt Lake City, UT, 84112, USA

The integral Chow ring of toric Deligne–Mumford stacks

It is well-known that the integral Chow ring of a smooth toric variety is isomorphic to the “Stanley–Reisner” ring of its fan. Iwanari generalized this result to any toric orbifold, i.e., the integral Chow ring of a toric orbifold is isomorphic to the “Stanley–Reisner” ring of the corresponding orbifold fan.

Generalizing the quotient construction of simplicial toric varieties by D. Cox, Borisov, Chen and Smith defined toric Deligne–Mumford stacks. We prove that toric Deligne–Mumford stacks in the sense of Borisov–Chen–Smith can be constructed from toric orbifolds by taking roots of line bundles over the toric orbifolds. Using this result we compute the integral Chow ring of toric Deligne–Mumford stacks.

KIUMARS KAVEH, University of Toronto, Toronto, ON

Convex bodies, isoperimetric inequality and degree of line bundles

We show how to associate a convex body (i.e., a compact connected convex set in R^n) to a finite dimensional subspace of regular functions L on a (quasi) affine variety X (of $\dim n$) so that the volume of this convex set is responsible for the number of solutions of a generic system of equations $\{f_i = 0\}$ in L .

Same construction works to associated a convex body to an ample line bundle on a projective variety. The volume of the convex body is then responsible for the degree of the line bundle. This can be regarded as a generalization of the Newton polytopes and the well-known Kushnirenko theorem in toric geometry as well as the Gelfand–Cetlin polytopes in representation theory. This will have several interesting applications, e.g. the well-known Brunn–Minkowskii inequality (generalization of isoperimetric inequality) in convex geometry gives Hodge index theorem and log-concavity of the degree function of line bundles.

DANIEL KRASHEN, University of Pennsylvania

Index reduction formulas for Brauer classes

One of the central problems in the study of division algebras is to determine the dimension of a given division algebra given its Brauer class. In the problem of index reduction, one starts with a division algebra on a field F , a field extension L/F and asks how to compute the dimension of the division algebra on L corresponding to the pullback of the original Brauer class on F .

In this talk I will discuss the use of stable twisted sheaves for computation of the index of a Brauer class, and in particular the case when L is the function field of a curve of genus 1 over F .

This is joint work with M. Lieblich.

MANISH KUMAR, Michigan State University, Wells Hall, East Lansing, MI 48824

The fundamental group of smooth affine curves in positive characteristic

The étale fundamental group of smooth curves defined over complex numbers (or any algebraically closed field of characteristic 0) are easy to compute thanks to topology and Riemann’s existence theorem. But the scenario is very different if the

characteristic is nonzero. For instance, the fundamental group of a smooth affine curve in positive characteristic is not even (topologically) finitely generated. This is because in positive characteristic one could have many types of wild ramifications at the boundary. In fact, in this talk we shall see if the base field is countable algebraically closed field then the commutator subgroup of the étale fundamental group of any smooth affine curve turns out to be profinite free group over countably many variables. The proof is Galois theoretic and uses formal patching techniques.

BEHRANG NOOHI, Florida State

GREG SMITH, Queen's University
Constructions of toric Deligne–Mumford stacks

In this talk, we will illustrate how various groupoid presentations of a toric Deligne–Mumford stack are related to different cohomology rings for the stack.

RAZVAN VELICHE, University of Utah, Salt Lake City, Utah
Maximally symmetric stable curves

In joint work with Michael van Opstall, we give a sharp bound on the number of automorphisms of a stable curve of a given genus and describe all curves attaining this bound.