
MIKE FRIED, UC Irvine

Atomic Orbital-type cusps on Alternating Group Modular Towers

Reduced Hurwitz spaces—spaces of r -branched Riemann surface covers of the projective line—are dimension $r - 3$ moduli spaces. These stacks have cusps on their boundaries. They can have fine moduli, but often do not. In the form of **M(odular) T(owers)** they support conjectures generalizing modular curve statements. Other researchers use these to connect the *Inverse Galois Problem* and the *Strong Torsion Conjecture* (on abelian varieties).

Like Shimura varieties—some are special cases—each **MT** comes with a prime p . As many **MTs** attach to p as there are p -perfect finite groups. We get a hold on these spaces using a *sh-incidence pairing* on their cusps.

We will concentrate on applying the sh-incidence pairing to infinitely many **MTs** where the Main Conjectures are proved. We chose examples of Liu and Osserman, who proved a first connectedness result. Here the projective line covers have alternating groups as monodromy groups, $p = 2$ and $r = 4$ (so tower levels are upper half plane quotients, but not modular curves). I swear, the group theory is surprisingly easy.

We use a “Fried–Serre” spin-lifting formula to locate 2 cusps. Our computations were guided by the look of an *atomic orbital* in sh-incidence rows. By catching 2 cusps at tower level 1—though there are none at level 0—we prove the Main Conjecture. We end by comparing p cusps in **MTs** with those of modular curve towers.