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Construction of compactly supported wavelets adapted to differential operators

We present constructive methods to obtain compactly supported biorthogonal wavelets adapted to differential (pseudodifferential) operators. Such wavelets provide a diagonal form of the corresponding operators and can be useful for numerical applications. Note that the construction of the wavelets boils down mainly to obtain the corresponding masks. Note also that operator-adapted wavelets are closely connected with wavelet bases for some functional spaces (in particular, Sobolev's spaces). We introduce an original approach to obtain wavelets on an interval adapted to monomial differential operators. A generalization to sum of differential operators is discussed. We present a method to adapt wavelets to differential operators with polynomial coefficients. In the multidimensional case we introduce a construction of wavelets adapted to hyperbolic differential operators. A preliminary idea on how to adapt wavelets to the Laplace operator is also considered. Note that, in general, operator-adapted wavelets are not shift and scale invariant (second generation wavelets) and this raises the problem of the well-posedness of the corresponding Multiresolution Analyses. We also discuss a conceptual possibility of adapting frames to operators.