Mathematical Aspects of Continuum Physics: Analysis, Computation, and Modeling
Aspects mathématiques de la physique du continu: analyse, analyse computationnelle et modélisation

(Org: **Rustum Choksi** (SFU) and/et **Mary Pugh** (Toronto))

STAN ALAMA, McMaster University, Hamilton, Ontario, Canada

Vortices in multiply connected Bose–Einstein condensates

We study minimizers of the Gross–Pitaevskii energy, introduced to model Bose–Einstein condensates (BEC) which are subject to a uniform rotation. This energy is very closely related to the Ginzburg–Landau energy of superconductivity, and an essential feature of the model is the formation of quantized singularities (vortices) in an appropriate singular limit. Following some recent experiments in BEC, we consider condensates with annular (planar) or toroidal (3D) geometry and examine minimizers to determine the presence and location of vortices in the condensate as the rotational speed increases. These questions involve singularly perturbed elliptic systems, and we will use variational methods with sharp estimates on the energy together with some tools from geometric measure theory to study the 3D case.

These results have been obtained in collaboration with L. Bronsard, A. Aftalion and J. A. Montero.

FERNANDO BRAMBILA, Departamento de Matemáticas, Facultad de Ciencias, UNAM, México

Complete Vectorial Radon Transform

We will talk about the complete inversion formula for the Vectorial Radon Transform.

It is well known how to recover the curl of a vector field using Doppler effect (G. Sparr, 1995). Using more information and Helmholtz, it is possible to have a complete inversion.

IRENE FONSECA, Department of Mathematical Sciences, Carnegie Mellon University, Pittsburgh, PA 15213, USA

Surfactants in Foam Stability: a Phase Field Model

The role of surfactants in stabilizing the formation of bubbles in foams is studied using a phase-field model. The analysis is centered on a van der Walls–Cahn–Hilliard-type energy with an added term accounting for the interplay between the presence of a surfactant density and the creation of interfaces. In particular, it is concluded that the surfactant segregates to the interfaces, and that the prescription of the distribution of surfactant will dictate the locus of interfaces, which is in agreement with experimentation.

This is joint work with Massimiliano Morini and Valeriy Slastikov.

CARLOS GARCIA-CERVERA, Mathematics Department, University of California, Santa Barbara, CA 93106, USA

An Efficient Real Space Method for Orbital-Free Density Functional Theory

I will describe an efficient implementation of the truncated-Newton method for energy minimization in the context of orbital-free density functional theory. I will illustrate the efficiency and accuracy of the method with numerical simulations in an aluminium FCC lattice.

JOY KO, Brown University, Providence, Rhode Island, USA

Steady rotational water waves near stagnation

Two-dimensional finite-depth periodic water waves with general vorticity and large amplitude are computed. The mathematical formulation and numerical method that allow us to compute a continuum of such waves with arbitrary vorticity are described. The computations in the case of constant vorticity show that there are only two points of stagnation and that the qualitative nature of the free surface depend on the vorticity. For variable vorticity, modelling surface shears and undertows, phenomena such as internal stagnation can occur.

This is joint work with Walter Strauss (Brown University).

JONATHAN MATTINGLY, Department of Mathematics, Duke University, Durham, NC
Challenges in the analysis of degenerately forced stochastic PDEs

I will give a few example of degenerately forced PDEs which might be of modeling interest, such as reaction diffusion or fluid equations. Then I will discuss the mathematical difficulties which arise in the stochastic PDE setting which do not arise for stochastic ODEs. Then I will give some cases where we can circumvent these difficulties, either by direct calculation or by some new mathematical tools.

ROBERT MCCANN, University of Toronto Mathematics
Nonlinear diffusion from a delocalized source: affine self-similarity, spacetime asymptotics, and focusing geometry

A family of explicit solutions to the porous medium equation and its fourth order generalizations is described, in the full range of nonlinearities, in which the pressure is given by a quadratic function of space at each instant in time. These include spreading solutions whose source is concentrated on any conic region of dimension lower than the ambient space, and solutions which focus at conic regions. The singular limiting distributions are affine projections of Barenblatt type profiles with arbitrary signature. A time-reversal symmetry is revealed which transforms spreading solutions to focusing solutions, and vice versa. This yields new information about the long and short time asymptotics of finite-mass solutions, about the instability of focusing, and about singularity geometry.

This work is joint with Jochen Denzler (University of Tennessee at Knoxville). Preprints are found at `www.math.toronto.edu/mccann`.

GOVIND MENON, Brown University
Domain coarsening in a 1D bubble bath

We will study a mean-field model for coarsening in a 1D bubble bath. An explicit solution formula found by Gallay and Mielke allows us to provide a simple and complete characterization of the approach to self-similarity in this model.

This is work with Barbara Niethmammer and Bob Pego.

BOB PEGO, Carnegie Mellon Univ., Dept. Math. Sciences, Pittsburgh, PA 15213
Scaling dynamics of coagulation equations with dust and gel

We study limiting behavior of rescaled size distributions that evolve by Smoluchowski's rate equations for coagulation, with rate kernel $K = 2$, $x + y$ or xy. We find that the dynamics naturally extend to probability distributions on the positive half-line with zero and infinity appended, representing populations of clusters of zero and infinite size. The "scaling attractor" (set of subsequential limits) is compact and has a Levy–Khintchine-type representation that linearizes the dynamics and allows one to establish several signatures of chaos. In particular, for any given solution trajectory, there is a dense family of initial distributions (with the same initial tail) that yield scaling trajectories shadowing the given one for all large time.

MARIA G. REZNIKOFF, School of Mathematics, Georgia Institute of Technology, 686 Cherry Street, Atlanta, Georgia 30332
Slow Motion of Gradient Flows

Sometimes physical systems exhibit "metastability," in the sense that states get drawn toward so-called metastable states and are trapped near them for a very long time. A familiar example is the one-dimensional Allen–Cahn equation: Initial data is drawn quickly to a "multi-kink" state and the subsequent evolution is exponentially slow. The slow coarsening has been analyzed by Carr & Pego, Fusco & Hale, Bronsard & Kohn, and X. Chen.

In general, what causes metastability? Our main idea is to convert information about the energy landscape (statics) into information about the coarsening rate (dynamics). We give sufficient conditions for a gradient flow system to exhibit metastability. We then apply this abstract framework to give a new analysis of the 1-d Allen–Cahn equation. The central ingredient is to establish a certain nonlinear energy–energy–dissipation relationship. One benefit of the method is that it gives a natural proof of the fact that exponential closeness to the multi-kink state is not only propagated, but also generated.

This work is joint with Felix Otto, University of Bonn.

SILVIA SERFATY, New York University
The Ginzburg–Landau energy close to the second critical field

We are interested in the Ginzburg–Landau energy when the applied magnetic field approaches the "second critical field" from below. Then, bulk-superconductivity decreases and vortex lattices are expected. I will present joint results with Etienne Sandier/Amandine Aftalion, where we derive the uniform repartition of the energy and a limiting problem.

THOMAS WANNER, George Mason University, Department of Mathematical Sciences, Fairfax, VA 22030, USA
Complex Transient Patterns and their Topology

Many partial differential equation models arising in applications generate complex time-evolving patterns which are hard to quantify due to the lack of any underlying regular structure. Such models may include some element of stochasticity which leads to variations in the detail structure of the patterns and forces one to concentrate on rougher common geometric features. In many of these instances, one is interested in the geometry of sublevel sets of a function in terms of their topology, in particular, their homology. In practice, however, these sublevel sets are approximated using an underlying discretization of the considered partial differential equation—which immediately raises the question of the accuracy of the resulting homology computation. In this talk, I will present a probabilistic approach which gives insight into the suitability of this method in the context of random fields. We will obtain explicit probability estimates for the correctness of the homology computations, which in turn yield *a priori* bounds for the suitability of certain grid sizes. In addition, we present a computational approach to homology validation in the above setting, and apply our results to certain stochastic partial differential equations arising in materials science.