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Advances in homotopy methods for solving systems of complex polynomial equations

Let f be a system of polynomial equations, with complex coefficients. An approximate zero of f is a point z such that the Newton iterates, starting at z, converge quadratically to an exact zero of f. One of the most popular methods to find an approximate zero of f is known as homotopy deformation: Let g be another system which has a known solution ζ_0, and let $[g, f]$ be the segment joining these two systems. Under some (soft) hypotheses, there exists a curve of pairs system-solution $\Gamma_{\zeta_0}(t)$ such that $\Gamma_{\zeta_0}(0) = (g, \zeta_0)$ and $\Gamma_{\zeta_0}(1) = (f, \zeta)$, where ζ is a solution of f. Consider a partition $h_0 = g, \ldots, h_k = f$ of $[g, f]$, for some positive integer k. Then, Newton's method may be used to (closely) "follow" the curve of solutions $\Gamma_{\zeta_0}(t)$, so we obtain a final point z_k (close to ζ_k) that may be an approximate zero of the input system f.

The information we need to perform this homotopy deformation is the initial pair (g, ζ_0) and the number of intermediate steps $k > 0$ (which is the main ingredient for the complexity of the algorithm).

In this talk we will present a probabilistic method designed to find an initial pair (g, ζ_0) satisfying the following, for every $\varepsilon > 0$:

> *A randomly chosen input system f (fixed degrees and number of unknowns) is solved by the homotopy with initial pair (g, ζ_0) and k_ε steps, with probability $1 - \varepsilon$.*

Here, k_ε is a quantity satisfying the following inequality:

$$k_\varepsilon \leq p\varepsilon^{-1},$$

where p depends polynomially on the size of the input. We conclude that, if we admit a small probability of failure, one can find approximate zeros of systems in polynomial time.