DIMITRI SHLYAKHTENKO, University of California at Los Angeles L^2 Invariants, Free Probability and Operator Algebras

The Cheeger–Gromov L^2 Betti numbers of a discrete group are numerical invariants, going back to Atiyah's work on the equivariant Atiyah–Singer index theorem. On the other hand, Voiculescu has introduced another discrete group invariant, coming from his free probability theory, called the free entropy dimension. Very roughly, this number measures the "asymptotic dimensions" of the sets of approximate embeddings of a group into unitary matrices. We describe our joint work with A. Connes and I. Mineyev that has provided a connection between these numbers. Both of these approaches (operator algebra versions of L^2 Betti numbers and Voiculescu's free entropy dimension) are attempts to arrive at the von Neumann algebra analog of the theory of L^2 Betti numbers (Gaboriau) and of free entropy dimension for measurable equivalence relations.