
MAHIR HASANOV, Istanbul Technical University, Faculty of Science and Letters, Dept. of Mathematics, 34469 Maslak
Istanbul, Turkey

On some classes of nonlinear equations in Hilbert spaces arising from the spectral theory

Let $L(\alpha)$ be an operator function of the class $C^1([a, b], S(H))$ such that $L(a) \ll 0$, $L(b) \gg 0$ and for all $x \in H \setminus \{0\}$ the function $(L(\alpha)x, x)$ has exactly one zero $p(x)$ in (a, b) . Define the following nonlinear operator

$$Tx = \begin{cases} L(p(x))x, & x \neq 0, \\ 0, & x = 0. \end{cases}$$

We study a connection between solvability problems for the equation $Tx = y$ and completeness problems for eigenvectors of the operator functions $L(\alpha)$. We give some sufficient conditions for completeness of eigenvectors corresponding to eigenvalues from the interval $[a, b]$ which are based on separation properties of the functional $p(x)$.