Applied Partial Differential Equations
Équations différentielles appliquées

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JOE BIELLO, University of California, Department of Mathematics, Kerr Hall, One Shields Ave, Davis, CA 95616, USA

Rossby wave interaction between the tropics and midlatitudes: a novel asymptotic theory and solitary waves

Simplified asymptotic equations are developed for the non-linear interaction of long wavelength equatorial Rossby waves and barotropic Rossby waves with a significant midlatitude projection in the presence of suitable horizontally and vertically sheared zonal mean flows. The simplified equations allow for non-linear energy exchange between the barotropic Rossby waves and the baroclinic equatorial waves for non-zero zonal mean vertical shear through wave-wave interactions. Idealized examples in the model demonstrate that midlatitude Rossby wave trains in a baroclinic mean shear can transfer their energy to localized equatorially trapped baroclinic Rossby waves through a non-linear "westerly wind burst" mechanism. Conversely, equatorially trapped baroclinic Rossby wave trains in the idealized model can transfer substantial energy to the midlatitude barotropic Rossby waves. From the viewpoint of applied mathematics, the asymptotic equations derived here have several novel features. In particular, they admit analytic solitary wave solutions which correspond to interesting localized waves in the equatorial troposphere.

ANNE BOURLIOUX, Université de Montréal

Modelling error analysis and estimation for unsteady flamelets

Flamelet models are asymptotic solutions of the nonlinear PDEs used to simulate turbulent flames. They capitalize on the scale separation between the flame thickness (assumed to be thin) and the other flow scales by replacing the distorted, unsteady flame front by a one-dimensional, steady object. This simplifies the computations tremendously. An idealized set-up is used to systematically investigate this type of strategy, using a combination of asymptotic analysis and numerical simulations.

The class of unsteady flows under consideration consists of a shear with a time-modulated cross-flow. It is known to lead to very interesting intermittent mixing regimes for non-reactive problems. In the present reactive case, it has the property that, to leading order, the flame indeed matches a flamelet structure, but with a highly non-trivial (local and unsteady) dissipation, which is the key fitting parameter of the model. Different strategies to predict this parameter are examined and their regimes of validity are identified. One goal is to design an automatic procedure that would select the appropriate model as the computation proceeds. This requires an efficient modelling error estimation procedure. Preliminary efforts in that direction will also be discussed.

Joint work with Oleg Volkov.

ALEXEI CHEVIAKOV, University of British Columbia, Vancouver, BC

Framework for nonlocally-related PDE systems and nonlocal symmetries: Algorithmic approach

For a given PDE system, one can construct extended hierarchies ("trees") of nonlocally-related PDE systems. Each system in an extended tree is equivalent, in the sense that the solution set for any system in a tree can be found from the solution set for any other system in the tree. Due to the equivalence of solution sets, any coordinate-independent method of analysis (qualitative, numerical, perturbation, *etc.*) can be applied to any system within the tree, and may yield simpler computations and new results that cannot be obtained when the method is directly applied to the given system. Nonlocal symmetries and new local and nonlocal conservation laws for a given PDE system can arise from any system in its extended tree.

The concept of useful conservation laws plays an essential role in the construction of an extended tree. Useful conservation laws yield potential variables and equivalent nonlocally-related potential systems and subsystems for any given system.

We construct extended trees for the systems of Planar Gas Dynamics and Nonlinear Telegraph equations. Using the described framework, we demonstrate a direct relation between Eulerian and Lagrangian descriptions of gas dynamics, and find new families of conservation laws and new nonlocal symmetries.

The presented research was done in collaboration with George Bluman (UBC).

AMIK ST CYR, National Center for Atmospheric Research, 1850 Table Mesa Drive, Boulder, CO 80305
Optimized Schwarz methods for high-order spectral elements

In this presentation, it is shown how a small modification of the RAS, MS and AS-aug preconditioners at the algebraic level, motivated by optimized Schwarz methods defined at the continuous level, leads to a significant reduction in the iteration count of the iterative Krylov solver. Numerical experiments on the modified Helmholtz equation using a model problem and a next generation spectral element general circulation model on the sphere, illustrate the effectiveness of this new approach. Experimentally, it is observed that the best condition number attainable in 2D (without coarse solver), for a non-overlapping decomposition, is $\sqrt{N}$ where N is the order of the polynomial basis employed. The performance of the method on the Blue gene/L supercomputer is investigated.

Collaborators: Martin J. Gander and Stephen J. Thomas.

REINHARD ILLNER, University of Victoria
Entropy methods for degenerate drift-diffusion equations

I will present selected results from recent joint work with Jean Dolbeault, Philippe Bartier and Michal Kowalczyk. Linear drift-diffusion equations with degenerate or time-dependent coefficients arise in various applications, for example in traffic flow models or in flashing ratchet models. In more difficult (yet practically relevant) cases the coefficients may be coupled to moments of the dependent variable, producing a nonlinear problem. Entropy-entropy production estimates offer natural ways to describe the asymptotic behaviour of solutions to such problems, and I will show some of the relevant estimates.

NICOLAS KEVLAHAN, McMaster University
Multiscale space-time adaptive simulation of 2D turbulence

A space-time adaptive wavelet collocation method is developed to efficiently simulate two-dimensional incompressible turbulence. This new DNS technique takes advantage of the spatial and temporal intermittency of turbulence to approximate the solution in the space-time domain using an adaptive collocation wavelet method. Both spatial and temporal resolution are adapted locally to solve the vorticity equation to the desired tolerance. Note that the global time integration error is controlled: this is not possible using conventional time marching methods. We will present results for the merging of identical vortices at Re $= 1000$, and for decaying two-dimensional turbulence. We find that the total number of active space-time degrees of freedom is significantly smaller than in a conventional dynamically adaptive time marching method. We also expect to present an estimate of the number of space-time degrees of freedom for decaying 2D turbulence as a function of Reynolds number.

BOUALEM KHOUIDER, University of Victoria, 3800 Finnerty Road, Victoria, BC
Multicloud parametrizations for convectively coupled tropical waves

The tropical large scale circulation has a significant impact on our weather and climate through various meteorological disturbances originating near the equator such as El Niño, the Madden–Julian Oscillation, and various convectively coupled tropical waves. Such disturbances are often a result of organized tropical convection over a large range of scales; from the single clouds of 1 to 10 km to cloud clusters and superclusters of a few hundred to a few thousand kilometers. However, today's general circulation models (GCMs) perform very poorly in predicting/representing these phenomena because on the one hand they occur at the sub-grid scale for the GCMs and on the other hand the underlying physics are still not completely understood—impossible to parametrize accurately.

Recent observational analysis reveals the central role of three multi-cloud types, congestus, stratiform, and deep convective cumulus clouds, in the dynamics of large scale convectively coupled Kelvin waves, westward propagating two-day waves, and the Madden–Julian oscillation. We present in this talk a systematic model convective parametrization highlighting the dynamic role of the three cloud types through two baroclinic modes of vertical structure: a deep convective heating mode and a second mode with low level heating and cooling corresponding respectively to congestus and stratiform clouds.

Joint with A. J. Majda (NYU).

HORST LANGE, Mathematisches Institut, Universität Köln, Weyertal 86–90, D-50931 Köln, Germany

On the controllabilty of nonlinear Schrödinger equations

We consider the controllabilty of nonlinear Schrödinger equations in two specific cases, namely the nonlinear Hartree equation (of quantum chemistry), and the Gross–Pitaevskii equation (in the theory of Bose–Einstein condensation). We study the mathematical structure of the sets of reachable and nonreachable states, and show, *e.g.*, that the set of nonreachable states for the Hartree equation is "fat" in the Baire categorical sense, and dense in state space, whereas the set of reachable states is "meagre". In the Gross–Pitaevskii case the nonreachable states form a finite dimensional manifold.

Joint work with Reinhard Illner, Victoria, and Holger Teismann, Acadia.

ADAM MONAHAN, School of Earth and Ocean Sciences, University of Victoria

Covariance Structure of a Fluctuating Midlatitude Jet

Principal Component Analysis (PCA) is a standard technique used in ocean/atmosphere physics to look for structure in large multivariate datasets; mathematically, PCA involves finding the eigenstructure of the covariance matrix. Individual PCA basis functions are often assumed to represent distinct physical "modes" of variability. In this talk, we will develop analytic expressions for the covariance structure of an idealised midlatitude jet that can vary in strength, width, and position. Through a systematic perturbation analysis, we can read off the leading few eigenvectors (PCA modes) of the covariance matrix.

This analysis demonstrates that even in this idealised system, many of the assumptions commonly made in interpreting PCA structures are false. In particular:

(1) the PCA time series are uncorrelated, but not independent,

(2) individual PCA "modes" do not represent individual physical processes, and

(3) PCA structures arising due to individual processes alone can be mixed, or "hybridised", when these processes occur simultaneously.

ADAM OBERMAN, SFU

Numerical approximation of first and second order non-linear elliptic PDEs

The theory of viscosity solutions gives powerful existence, uniqueness and stability results for first and second order degenerate elliptic equations. The approximation theory developed by Barles and Sougandis in the early nineties gave conditions for the convergence of numerical schemes.

Building on this work, we develop convergent schemes for non-linear second order equations, including: infinity laplacian, motion by mean curvature, the Monge–Ampere equation.

We'll also discuss adaptive schemes on unstructed grids for first order equations. A motivating example is the high-dimensional control problem of airplane flight.

VLADISLAV PANFEROV, McMaster University

Global regularity of one-dimensional solutions of the Boltzmann equation

For the Boltzmann equation the setting of a narrow shock tube implies that solutions depend only on one spatial coordinate, while having a three-dimensional velocity dependence. We study the propagation of some regularity estimates, such as sup-norms of the macroscopic density, for the corresponding solutions of the Boltzmann equation. Using the methods based on the relative entropy control and on a certain nonlinear functional introduced by Bony and Cercignani, we establish the global in time existence of regular solutions for some model cases of particle interactions.

Joint work with A. Biryuk and W. Craig.

OLIVIER PAULUIS, New York University

Toward the end of cumulus parameterization

The current generation of General Circulation Models can simulate the global atmosphere with a resolution of the order of 50 km. Such a resolution is insufficient to explicitly simulate the deep convective motions that are responsible for the bulk of the vertical energy transport in the atmosphere. To compensate for this limitation, climate models have used various cumulus parameterizations to account for the effects of convective motions on the atmospheric temperature, humidity and cloud distribution. However, due to the continuous increase in computational power, the next generation of global models might be run at a sufficiently fine resolution as to make the use of cumulus parameterization unnecessary.

This paper investigates the impacts of horizontal resolution on the statistical behavior of convection. An idealized radiative-convective equilibrium is simulated for model resolutions ranging between 2 and 50 km. The simulations are compared based upon the analysis of the mean state, of the energy and water vapor transport, and of the probability distributions functions for various quantities. It is found that, despite some bias in temperature and humidity, coarse resolution simulations are able to reproduce reasonably well the statistical properties of deep convective towers. This is particularly apparent in the cloud ice and vertical velocity distributions that exhibit a very robust behavior at resolution up to 16 km.

A theoretical scaling for the vertical velocity as function of the grid resolution is derived, based upon the behavior of an idealized air bubbles. The vertical velocity of an ascending bubble is determined by its aspect ratio, with a wide, flat parcel rising at a much slower pace than a narrow one. This theoretical scaling law can explain the behavior of the numerical simulations, and be used to re-normalize the probability distribution functions for vertical velocity.

FRANCIS POULIN, University of Waterloo

Turbulent self-diffusion in isopycnal coordinates

A recent paper by Dukowicz and Smith (1997), henceforth referred to as [DS97], extends the classical theory of turbulent transport of a tracer particle (Morin and Yaglom, 1987) to encompass the problem of self-diffusion in stratified mesoscale oceanic turbulence, thereby shedding light onto the mathematical status and physical meaning of the recent parametrization of Gent and McWilliams (1990). This is interesting and important in view of the fact that the theory of geostrophic turbulence is still in its infancy.

The stated objective of [DS97] is to develop the stochastic theory of turbulent diffusion from the standard Fokker–Planck equation (Morin and Yaglom, 1987; Gardiner, 2004) in such a way that it also applies to compressible flow. The reason why this is deemed necessary is that when the classical, incompressible Boussinesq equations are expressed in isopycnal coordinates the velocity field ceases to be solenoidal. We will show that the argument presented in [DS97] is incorrect, although their main result can, fortunately, be salvaged.

STEVEN RUUTH, Simon Fraser University, Burnaby, BC V5A 1S6
Threshold Dynamics for Willmore Flow

Many important models of image processing and computer vision involve curvature-dependent functionals. The minimization of these functionals can involve the solution of fourth order geometric PDEs. The numerical solution of such PDEs with standard methods can be very costly.

Recently, Grzibovskis and Heintz (2005) proposed a threshold dynamics algorithm that approximates the gradient flow for an important curvature dependent functional known as the Willmore energy. This energy consists of the integral of the square of a surface's mean curvature over that surface. Furthermore, it constitutes an essential part of certain variational image models for segmentation with depth, disocclusion, and image inpainting. This talk discusses our recent work on practical threshold dynamics algorithms for Willmore Flow and the application of these algorithms to higher order models of image processing and computer vision.

This is joint work with Selim Esedoglu and Richard Tsai.

JOHN SCINOCCA, CCCma, MSC, University of Victoria
Wave Forcing, Parameterization, and the Breakdown of Newton's Third Law

Current parameterizations of gravity-wave drag (GWD) in general circulation models (GCMs) of the Earth's atmosphere explicitly conserve wave pseudomomentum flux and, therefore, satisfy Newton's Third Law. This approach assumes a basic-state flow that is horizontally uniform and it allows a direct connection between wave dissipation and wave-induced forcing of the flow. When the horizontal structure of the basic-state flow is no longer uniform this approach fails. In this instance the more fundamental principle of wave action conservation must be invoked. In this more general framework one can no longer associate all wave-induced forcing with wave dissipation. Newton's Third Law may be violated and when it is, the basic-state flow will be subjected to wave induced forces arising from wave dynamics that are conservative rather than dissipative in nature (Buhler and McIntyre 2005). In this study we reformulate a current parameterization of orographic GWD (Scinocca and McFarlane 2000) to allow horizontally non-uniform flow and to employ wave action flux, rather than pseudomomentum flux, as its primary conserved variable. The impact of this new formulation is investigated by offline calculations and fully interactive GCM simulations.

HOLGER TEISMANN, Acadia University, Wolfville, NS
Bilinear control of Schrödinger equations with confining potentials

We will discuss the control problem for linear and nonlinear Schrödinger equations with confining potentials, where the controls are given by applying spatially homogeneous fields. For quadratic potentials it can be shown that the equation is not controllable; the manifold of reachable states is finite-dimensional. On the other hand, K. Beauchard (2005) recently proved that the (linear) Schrödinger equation with an infinite square well (particle in a box) is controllable in the vicinity of the ground state. We will discuss some open problems and conjectures deriving from these observations.

VITALI VOUGALTER, University of Toronto, Department of Mathematics, 40 St. George Street, Toronto, ON M5S 2E4
Eigenvalues of zero energy in the linearized NLS problem

We study a pair of neutrally stable eigenvalues of zero energy in the linearized NLS equation. We prove that the pair of isolated eigenvalues of geometric multiplicity two and algebraic multiplicity $2N$ is associated with $2P$ negative eigenvalues of the energy operator, where $P = N/2$ if N is even and $P = (N-1)/2$ or $P = (N+1)/2$ if N is odd. When the potential of the linearized NLS problem is perturbed with a parameter continuation, we compute the exact number of unstable eigenvalues that bifurcate from the neutrally stable eigenvalues of zero energy.