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Concentration estimates for Emden–Fowler equations with boundary singularities and critical growth

We establish existence and multiplicity of solutions for the Dirichlet problem $\sum_i \partial_{ii} u + \frac{|u|^{2^\star - 2} u}{|x|^s} = 0$ on smooth bounded domains Ω of $\mathbb{R}^n$ $(n \geq 3)$ involving the critical Hardy–Sobolev exponent $2^\star = \frac{2(n-s)}{n-2}$ where $0 < s < 2$, and in the case where zero (the point of singularity) is on the boundary $\partial\Omega$. Just as in the Yamabe-type non-singular framework (*i.e.*, when $s = 0$), there is no nontrivial solution under global convexity assumption (*e.g.*, when Ω is star-shaped around 0). However, in contrast to the non-satisfactory situation of the non-singular case, we show the existence of an infinite number of solutions under an assumption of local strict concavity of $\partial\Omega$ at 0 in at least one direction. More precisely, we need the principal curvatures of $\partial\Omega$ at 0 to be nonpositive but not all vanishing. We also show that the best constant in the Hardy–Sobolev inequality is attained as long as the mean curvature of $\partial\Omega$ at 0 is negative. The key ingredients in our proof are refined concentration estimates which yield compactness for certain Palais–Smale sequences which do not hold in the non-singular case.

This is joint work with Frederic Robert.