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Eigenvalue Optimization, Spikes, and the Neumann Green's Function

An optimization problem for the fundamental eigenvalue of the Laplacian in a planar simply-connected domain that contains N small identically-shaped holes, each of a small radius $\epsilon \ll 1$, is considered. The boundary condition on the domain is assumed to be of Neumann type, and a Dirichlet condition is imposed on the boundary of each of the holes. The reciprocal of this eigenvalue is proportional to the expected lifetime for Brownian motion in a domain with a reflecting boundary that contains N small traps. For small hole radii ϵ, we derive an asymptotic expansion for this eigenvalue in terms of the hole locations and the Neumann Green's function for the Laplacian. For the unit disk, ring-type configurations of holes are constructed to optimize the eigenvalue with respect to the hole locations. For a one-hole configuration, the uniqueness of the optimizing hole location in symmetric and asymmetric dumbbell-shaped domains is investigated. This eigenvalue optimization problem is shown to be closely related to the problem of determining certain vortex configurations in the Ginzburg–Landau theory of superconuctivity and to the problem of determining equilibrium locations of particle-like solutions, called spikes, to certain singularly perturbed nonlinear reaction-diffusion systems. Some results for the equilibria, stability, and bifurcation behavior, of these spike solutions are given.