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The t-improper chromatic number of random graphs

We consider the t-dependence and t-improper chromatic numbers of the Erdős–Rényi random graph. As usual, $G_{n,p}$ denotes a random graph with vertex set $[n] = \{1, \dots, n\}$ in which the edges are chosen independently at random with probability p. The t-dependence number $\alpha^t(G)$ of a graph G is the maximum size of a t-dependent set—a vertex subset which induces a subgraph of maximum degree at most t. The t-improper chromatic number $\chi^t(G)$ is the smallest number of colours needed in a t-improper colouring—a colouring of the vertices in which colour classes are t-dependent sets. Note that $\chi^t(G) \geq |V(G)|/\alpha^t(G)$ and $\chi^t(G) \geq \chi^{t+1}(G)$ for any graph G.

Clearly, when $t = 0$, we are considering the independence number and chromatic number of random graphs, and the problem of determining the asymptotic behaviour of the chromatic number $\chi(G_{n,p})$ was once one of the central open problems in random graph theory. We consider a fixed edge-probability p where $0 < p < 1$, and $t = t(n)$. Our results on $\chi^t(G_{n,p})$ break into three ranges for $t(n)$. We say that a property holds asymptotically almost surely (a.a.s.) if it holds with probability $\to 1$ as $n \to \infty$.

(a) If $t(n) = o(\log n)$, then $\chi^t(G_{n,p}) \sim (\frac{1}{2} \log \frac{1}{1-p}) \frac{n}{\log n}$ a.a.s. Thus, if the impropriety t does not grow too fast, then the t-improper chromatic number is likely to be close to the proper chromatic number.

(b) If $t(n) = \Theta(\log n)$, then $\chi^t(G_{n,p}) = \Theta(\frac{np}{t})$ a.a.s.

(c) Lastly, if $t(n)/\log n \to \infty$, then $\chi^t(G_{n,p}) \sim \frac{np}{t}$ a.a.s.

This is joint work with Colin McDiarmid.