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*Ordered $k$-colorings dichotomy*

We introduce three variants of proper colorings with imposed partial ordering on the set of colors and will present dichotomy theorems that separate these problems into *tractable* and *NP-complete*.

Vertices of all considered graphs are integers from 1 to $|V(G)|$, hence they form a linearly ordered set $(V(G), \leq)$. The set of vertices colored $c$ will be denoted by $V_c$. Given a partially ordered set (poset) $(\mathcal{C}, \preceq)$ of colors, in the first problem we want to (properly) color vertices of $G$ by colors in $\mathcal{C}$ (*color $G$ by poset $\mathcal{C}$*) such that for any two colors $c$ and $c'$ if $c \preceq c'$ then for any two vertices $u \in V_c$ and $v \in V_{c'}$, $u \leq v$. Thus, if $\preceq$ is the empty relation on $\mathcal{C}$, then the problem is whether $G$ can be properly colored with $|\mathcal{C}|$ colors, a well known graph coloring problem.

In the second problem, we want to color $G$ by poset $\mathcal{C}$ such that for any two colors $c$ and $c'$ if $c \preceq c'$ then for any two adjacent vertices $u \in V_c$ and $v \in V_{c'}$, $u \leq v$. This problem is the well-known directed graph homomorphism problem whose dichotomy was extensively studied.

In the last problem, we want to color $G$ by poset $\mathcal{C}$ such that for any two colors $c$ and $c'$ if $c \preceq c'$ then for any two vertices $u \in V_c$ and $v \in V_{c'}$ in a component induced by $V_c \cup V_{c'}$, $u \leq v$.

This is a joint work with Arvind Gupta, Jan van den Heuvel, Jan Manuch, and Xiaohong Zhao.