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Silver Cubes

An $n \times n$ matrix is *silver* if, for $i = 1, \ldots, n$, every symbol in $\{1, 2, \ldots, 2n-1\}$ appears as an entry in either row i or column i.

An IMO 1997 question introduced this definition, and asked whether a silver matrix of order 1997 exists. (In fact, one exists if and only if $n = 1$ or n is even.) In this paper we investigate higher dimensional analogs, *silver cubes*.

Finding the correct generalization is the first challenge. The cells on the main diagonal of a silver matrix are treated specially. What should serve as a "diagonal" in a d-dimensional $n \times n \times \cdots \times n$ silver cube? We propose that a "diagonal" should be a "maximum independent set in the d-th cartesian power of the complete graph of order n." This definition is motivated by "minimal defining sets" for colouring such graphs. The challenge is to label the cells with symbols $1, 2, \ldots, d(n-1)+1$ so that, for each cell c on the "diagonal", every symbol appears once in one of the $d(n-1)+1$ cells orthogonally translated from c. We present constructions, nonexistence proofs and connections with coding theory and projective geometry.

This is joint work with M. Ghebleh, E. Mahmoodian and M. Verdian-Rizi.