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Degree Sequence Problems for 3-Hypergraphs

The Erdős–Gallai conditions are necessary and sufficient conditions for the existence of a simple graph with a given degree sequence. Much work has been done characterizing the polytope of degree sequences of simple graphs. The corresponding conditions for 3-hypergraphs are still unknown.

A simple 3-hypergraph G consists of a set V of vertices and E of edges, such that each edge is a triple u, v, w of distinct vertices. Repeated triples are not allowed in G. The degree of a vertex v is $\deg(v)$, the number of triples containing v. The degree sequence of G is the sequence of degrees $D(G) = (d_1, d_2, \ldots, d_n)$, such that $d_1 \geq d_2 \geq \cdots \geq d_n$. We ask when a given sequence D is the degree sequence of a simple 3-hypergraph?

It is still unknown whether this problem has a polynomial-time algorithmic solution, or whether it is NP-complete. Recently Kocay and Li showed that any two 3-hypergraphs with the same degree sequence can be transformed into each other by a sequence of operations known as trades. The proof is based on null-hypergraphs. We describe the structure of null-hypergraphs, and a closely related NP-complete problem for 3-hypergraph degree sequences.