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The number of minimum volume tetrahedra

Determining the maximum number of unit distances determined by n points in the plane is one of the notoriously hard Erdős problems in combinatorial geometry. It is easy to give a tight bound on number of occurrences of the minimum and maximum distance, which is at most $3n - O(\sqrt{n})$ and n, respectively. Finding the maximum number of unit area triangles determined by n points in the plane is similarly hard as the unit distance problem. It is known, however, that the minimum and maximum triangle areas can occur $O(n^2)$ and $O(n)$ times, and both bounds are tight. We pursue the analogous problems in the space, and find bounds on the maximal number of unit, minimum, and maximum volume tetrahedra determined by n points in three dimensions, along with some new techniques.