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Free Boundaries in Optimal Transport

Given a distribution f of iron mines throughout the countryside, and a distribution g of factories which require iron ore, the optimal transportation problem of Monge and Kantorovich asks for the mines to be paired with the factories so as to minimize the average (say) Euclidean distance squared between mine and factory. This problem is deeply connected to geometry, inequalities, and nonlinear differential equations, with applications ranging from shape recognition to weather prediction.

In the talk I discuss what happens when the production capacity of the mines need not agree with the demand of the factories, so one ships only a certain fraction of the ore being produced, again choosing the locations of factories and mines which remain active so as to minimize total transportation costs. If the mines are continuously distributed in Euclidean space, and positively separated from the factories, the solution will be unique, and is given by pair of domains $U, V \subset R^n$, with U containing the active mines and V the active factories. These domains are characterized as the non-contact regions in a double obstacle problem for the Monge–Ampère equation. We go on to specify conditions on f and g which are sufficient to ensure that U and V have continuously differentiable free boundaries, and that the correspondence $s\colon \overline{U} \longrightarrow \overline{V}$ mapping mines to factories is a homeomorphism or smoother, Hölder continuous up to the free (and part of the fixed) boundary.