

MEGUMI HARADA, University of California-Berkeley, California, USA

The symplectic geometry of the Gel'fand-Cetlin basis for representations of the symplectic group

The Gel'fand-Cetlin canonical basis for a finite-dimensional representation V_λ of $U(n, \mathbb{C})$ can be constructed by successive decompositions of the representation by a chain of subgroups

$$U(1, \mathbb{C}) \subset U(2, \mathbb{C}) \subset \cdots \subset U(n-1, \mathbb{C}) \subset U(n, \mathbb{C}).$$

A key point in the analysis is that the decomposition of an irreducible representation of $U(k, \mathbb{C})$ under the subgroup $U(k-1, \mathbb{C})$ is multiplicity-free. Guillemin and Sternberg constructed in the 1980s the Gel'fand-Cetlin integrable system on the coadjoint orbits \mathcal{O}_λ of $U(n, \mathbb{C})$, which is the symplectic geometric version, via geometric quantization, of the Gel'fand-Cetlin canonical basis for V_λ .

For $G = U(n, \mathbb{H})$, the compact symplectic group (also described as the *quaternionic* unitary group), however, the decompositions are not multiplicity-free. However, in recent years, Molev et al. have found a Gel'fand-Cetlin type basis for representations of the symplectic group, using essentially new ideas, including the Yangian $Y(2)$, an infinite-dimensional quantum group, and a subalgebra called the twisted Yangian $Y^-(2)$. In this talk I will explain the symplectic and Poisson geometry underlying the canonical basis for finite-dimensional irreducible representations of $U(n, \mathbb{H})$. In particular, I will construct an integrable system on the symplectic reductions of the coadjoint orbits of $U(n, \mathbb{H})$ by $U(n-1, \mathbb{H})$ and explain its correspondence with Molev *et al.*'s work.