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Purely nonlinear instability of minimal energy standing waves

For a variety of nonlinearities, the nonlinear Schroedinger equation is known to possess localized quasistationary solutions ("standing waves"). We prove that in the generic situation the standing wave of minimal energy among all other standing waves is unstable. This case was falling out of the scope of the classical paper by Grillakis, Shatah, and Strauss on orbital stability of standing waves. An interesting feature of the problem is the absence of (exponential) instability in the linearized system; in this sense, the resulting instability is "purely nonlinear". Essentially, the instability is caused by higher algebraic degeneracy of zero eigenvalue in the spectrum of the linearized system. The result can be generalized to abstract Hamiltonian systems with $U(1)$ symmetry.