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Centre-valued index of Toeplitz operators with noncommuting symbols

We begin with a unital C^*-algebra A and a unital C^*-subalgebra, Z of the centre of A. We assume that we have a faithful, unital Z-trace τ and a continuous action $\alpha\colon \mathbf{R} \to \mathrm{Aut}(A)$ leaving τ and hence Z invariant. We let δ be the infinitesimal generator of α on A.

We have in this setting a *largest* (in the sense of quasi-containment) $*$-representation of A on a Hilbert space which carries a *faithful*, unital u.w.-continuous $Z^{-u.w.}$-trace $\bar\tau\colon A^{-u.w.} \to Z^{-u.w.}$ extending τ. We assume that A is concretely represented on this Hilbert space. We denote by $\mathfrak{A}$ and $\mathfrak{Z}$ respectively, the ultraweak closures of A and Z. One shows that there is an u.w.-continuous action $\bar\alpha\colon \mathbf{R} \to \mathrm{Aut}(\mathfrak{A})$ extending α and leaving $\bar\tau$ and $\mathfrak{Z}$ invariant.

At this point we construct a representation, $\mathrm{Ind} = \tilde\pi \times \lambda$ of $A \rtimes \mathbf{R}$ on a certain self-dual Hilbert-$\mathfrak{Z}$ module $H_{\mathcal{A}}$ constructed from a certain "$\mathfrak{Z}$-Hilbert Algebra," $\mathcal{A}$. We let $\mathcal{M} = \mathrm{Ind}(A \rtimes \mathbf{R})''$ which contains $\mathfrak{Z}$ in its centre and has a faithful, normal semifinite $\mathfrak{Z}$-trace $\hat\tau$. *This construction is half the battle.* We let H denote the image of the Hilbert Transform in $\mathcal{M}$ and let $P = \frac{1}{2}(H+1)$ in $\mathcal{M}$. We then consider the semifinite von Neumann algebra,

$$\mathcal{N} := P\mathcal{M}P$$

with the faithful, normal, semifinite $\mathfrak{Z}$-trace obtained by restricting $\hat\tau$. For $a \in A$ we define the *Toeplitz* operator

$$T_a := P\tilde\pi(a)P \in \mathcal{N}.$$

We prove the following theorem.

Theorem 1 *Let A be a unital C^*-algebra and let $Z \subseteq Z(A)$ be a unital C^*-subalgebra of the centre of A. Let $\tau\colon A \to Z$ be a faithful, unital Z-trace which is invariant under a continuous action α of $\mathbf{R}$. Then for any $a \in A^{-1} \cap dom(\delta)$, the Toeplitz operator T_a is Fredholm relative to the trace $\hat\tau$ on $\mathcal{N} = P\big(\mathrm{Ind}(A \rtimes \mathbf{R})''\big)P$, and*

$$\hat\tau\text{-}\mathrm{ind}(T_a) = \frac{-1}{2\pi i}\tau\big(\delta(a)a^{-1}\big).$$