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Summatory functions of elements in Selberg's class

Let $F(s)$ be a Dirichlet series, $F(s) = \sum_{n=1} a_n n^{-s}$, $\Re s > 1$. Define the the summatory function $S(x)$ to be $\sum_{n \le x} a_n$. We assume that $F(s)$ satisfies the following conditions. First, for all $\epsilon > 0$, $|a_n| = O(n^\epsilon)$. In addition, it admits analytic continuations and functional equations. More precisely, there is a function $\Delta(s) = Q^s \prod \Gamma(\alpha_i s + \gamma_i)$, $Q > 0$, $\alpha_i > 0$, $\Re\gamma_i > 0$, such that $F(s)\Delta(s) = \omega \bar{F}(1-s)\bar{\Delta}(1-s)$, $|\omega| = 1$. Furthermore, assume that $F(s)$ is entire. Twice of the summation of α_i is called the *degree* d_F of F. In this talk, I will derive an estimation of $S(x)$ without extra conditions. The trivial estimation is $S(x) = O(x^{1+\epsilon})$, $\forall \epsilon > 0$.

I will provide two estimations of $S(x)$. One is a joint work with Ram Murty; we prove that for $d_F \ge 1$, $S(x) = O(Q^{1-\theta+\epsilon} x^{\theta+\epsilon})$, where $\theta = d/(d+2)$. For the larger $d_F \ge 2$, I can get a better result: $S(x) = O(Q^{1-\theta'+\epsilon} x^{\theta'+\epsilon})$, where $\theta = (d-1)/(d+1)$. In both cases, the implied constants are independent of Q.