

CHANTAL DAVID, Concordia University  
*On the vanishing of twisted  $L$ -functions of elliptic curves*

(joint work with J. Fearnley and H. Kisilevsky)

Let  $E$  be an elliptic curve over the rationals with  $L$ -function  $L_E(s)$ . Let  $\chi$  be a Dirichlet character, and let  $L_E(s, \chi)$  be the  $L$ -function of  $E$  twisted by the character  $\chi$ . For quadratic characters  $\chi$ ,  $L_E(1, \chi)$  vanishes for at least half of the characters (where the sign of the functional equation is  $-1$ ), and Goldfeld conjectured that the density of vanishing is exactly  $1/2$  in this case. For higher order characters, the functional equation now relates  $L_E(1, \chi)$  and  $L_E(1, \bar{\chi})$ , and there is no reason to predict a positive density of vanishing. We present in this talk some evidence for the case of twists by cubic character  $\chi$ , based on empirical computations and random matrix theory.