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The Schur-Horn Theorem in II_1 factors

The Schur-Horn theorem expresses a relation, in real n-space, of two apparently unrelated notions: majorization and convexity. In finite dimensional von Neumann algebras, it can be expressed in terms of maximal abelian subalgebras (masas) of factors:

Let M be a finite dimensional factor, $A \subset M$ a masa in M, and let P be the unique conditional expectation from M to A. Then, for every $a \in A$,

$$P\big(\mathcal{U}_M(a)\big) = \mathrm{co}\{vav^* : v \in \mathcal{N}(A)\},$$

where $\mathcal{U}_M$ is the unitary group of M and $\mathcal{N}(A)$ is the normalizer of A.

The author believes (and has evidence, although not a proof) that the Schur-Horn property characterizes masas of finite dimensional factors (that is, if a subalgebra of a finite dimensional factor satisfies the Schur-Horn relation, then it is a masa).

II_1 factors are a natural setting to try the Schur-Horn property, because there always exist a normal conditional expectation onto any masa, and also because the main notion used in the finite dimensional proof is majorization, which can be reasonably extended to the setting of II_1 factor, thanks to the existence of the trace. One cannot expect masas in II_1 factors to be characterized by the Schur-Horn property (it fails clearly for singular masas) but it may, for instance, characterize regular masas.

In our attempts to prove this theorem, we have had to work with the notion of majorization in II_1 factors, and several interesting characterizations appear for unitary orbits of states and automorphisms.