Automorphic forms and representations
Formes automorphes et représentations
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NICOLAS ARANCIBIA, Cergy Paris Université
Proving the Enhanced Shahidi Conjecture for Real Groups.

For a real group G, it follows from the work of Kostant and Vogan that the L-packet associated with an L-parameter ϕ contains a generic representation if and only if the corresponding $\check{G}$-orbit in the variety of geometric parameters is open.

In this talk, we prove an analogous criterion for Adams–Barbasch–Vogan micro-packets: the micro-packet attached to ϕ contains a generic representation precisely when the associated orbit is open. As an application, we establish the Enhanced Shahidi Conjecture for real groups, showing that the Arthur packet attached to an A-parameter ψ contains a generic representation if and only if the restriction $\psi|_{\mathrm{SL}_2}$ is trivial.

Micro-packets are defined in terms of characteristic cycles of D-modules. To establish our result, we explicitly compute these characteristic cycles in the case of D-modules associated with generic representations. We do this by exploiting the existence of a Weyl group action on both the domain and the codomain of the characteristic cycle map, which makes this map equivariant.

KRISTAPS BALODIS, University of Calgary
Arthur parameters and orbits with singular closure

Building on the work of Zelevinsky, and the cases for real and complex groups, David Vogan proposed a p-adic Kazhdan-Lusztig hypothesis: The dimensions of stalks of perverse sheaves on the "Vogan variety" V_λ of Langlands parameters having infinitesimal parameter λ, should coincide with multiplicities of irreducible representations of infinitesimal parameter λ in standard representations. Moreover, Vogan defined what we call ABV-packets in terms of the microlocal geometry of V_λ, and proposed that these coincide with Arthur's A-packets.

In this talk, I will discuss recent work with collaborators that, if G is a p-adic group for which the p-adic Kazhdan-Lusztig hypothesis holds, then the forward direction of the Gross-Prasad conjecture holds: An L-packet $\Pi_\phi(G)$ contains a generic representation if and only if $L(s, \phi, \mathrm{Ad})$ is regular at $s = 1$. I will also discuss an analogous result for ABV-packets, and implications for Shahidi's enhanced genericity. Finally, I will discuss some results towards showing that Arthur parameters of classical groups and $\mathrm{GL}(n)$ are associated to orbits of singular closure in their Vogan variety, and counter-examples for non-classical groups.

ADELE BOURGEOIS, Tutte Institute/Carleton University
Making sense of norm lifts for semisimple characters

Let G be a connected reductive group over a local nonarchimedean field F of residual characteristic p and set $H = (G^\Gamma)^\circ$, where $\Gamma \subset \mathrm{Aut}(G)$ is a finite group such that $\gcd(p, |\Gamma|) = 1$. We show how one can lift a semisimple character for H, in the sense of Bushnell-Kutzko, to a semisimple character for G. We also discuss how the norm map may be used to create a different type of lift.

JOSE CRUZ, University of Calgary
On the transcendental nature of the class number formula

A classical problem in number theory is how to determine if two number fields are isomorphic. One approach is to verify whether the given number fields have the same arithmetical invariants (e.g., discriminant, signature, regulator, among others). In this talk, I will focus on the regulator, and show how much arithmetical information it encodes. More specifically, I will give,

conditionally on the Weak Schanuel Conjecture, necessary and sufficient conditions for two totally real number fields to have the same regulator.

We will see that these conditions naturally appear from the following result: Two totally real number fields K_1 and K_2 have the same Dedekind zeta functions $\zeta_{K_1}(s)$ and $\zeta_{K_2}(s)$ if and only if the real numbers

$$\lim_{s\to 1}(s-1)\zeta_{K_1}(s) \quad \text{and} \quad \lim_{s\to 1}(s-1)\zeta_{K_2}(s)$$

are linearly dependent over the field of algebraic numbers $\overline{\mathbb{Q}}$. I will conclude with a representation-theoretic reformulation of this result.

JULIA GORDON, University of British Columbia

Coefficients of Harish-Chandra's local character expansion are motivic

This talk will be about the joint work with Loren Spice, where we prove that the coefficients of the Harsh-Chjandra's local character expansion for reductive p-adic groups are so-called motivic constants (I will define this notion). This leads to a uniform in p bound on the dimension of the space of fixed vectors of a family of representations of p-adic groups under a given (definable) compact subgroup.

SIMRAN KHUNGER, University of Michigan, Ann Arbor

Automorphic multiplicity formulas for inner forms of SL_n

In 2012, Hiraga and Saito proved Arthur's multiplicity formula for cuspidal automorphic representations of inner forms of SL_n, up to an indeterminate constant. Their method of proof utilized the stable trace formula, among other tools.

In this talk, we discuss a strategy to canonicalize Hiraga and Saito's result, effectively removing the indeterminate constant in their work. We use the example of $n = 2$ to illustrate this strategy.

ANDREW KNIGHTLY, University of Maine

Counting newforms with prescribed ramified supercuspidal components

We recently gave a general trace formula which counts the number of classical newforms of fixed weight and level whose local components at the primes dividing the level are prescribed supercuspidal representations. To make the formula explicit, one must compute certain local orbital integrals at each such prime. At present, this has been carried out only when the fixed supercuspidals have conductor exponent 2 or 3.

In joint work with Kimball Martin, we obtain a dimension formula that allows for prescribed supercuspidals of any odd-power conductor (the so-called ramified supercuspidals). We are able to get around having to compute the tricky orbital integrals by using the trace of an Atkin-Lehner operator. As a consequence of our formula, we observe a bias in global dimension formulas favoring certain supercuspidals over others with the same conductor exponent, though asymptotically there is balance as first shown in a much more general setting by Kim, Shin and Templier. Interestingly, the magnitude of this bias is independent of the (odd) conductor exponents. The sign of the bias is the global root number.

DIDIER LESESVRE, Université de Lille

Kuznetsov trace formula for GSp(4) and applications

Trace formulas relate statistics on automorphic forms, which often remain mysterious yet central in number theory, with statistics on geometric or arithmetic quantities, which one hopes to be more explicit and better understood. We will discuss how to establish such a Kuznetsov-type trace formula in the case of the symplectic group GSp(4), and will study the precise analytic behaviour of both the spectral and the arithmetic transforms arising in the formula. These fundamental properties can be used to establish various results on the family of Maaß automorphic forms on GSp(4) in the spectral aspect: the Weyl

law, a density result on the non-tempered spectrum, large sieve inequalities, bounds on the second moment of the spinor and standard L-functions, as well as a statement on the distribution of the low-lying zeros of these L-functions.

SPENCER LESLIE, Boston College
Endoscopic phenomena in relative Langlands

I will review ideas around the theory of endoscopy and the relative Langlands program. I will review several examples with applications to trace formula comparisons, sums of L-values, and harmonic-analytic anomalies on symmetric varieties.

CALLIE LIDDLE, Carleton University
Singularities of K-orbits on the flag variety of G_2

We define G_2 as a complex Lie group with Lie algebra of type G_2. K is the centralizer of a particular element of order 2 in G_2. We parameterize the orbits of K acting on the flag variety of G_2, and use MAGMA software to determine where the singularities of the K-orbit closures lie. We provide applications for the representation theory of G_2.

SADIE LIPMAN, University of Michigan
Toral supercuspidal representations and Shintani lifting

Given a tame elliptic regular pair $(S(F), \phi)$ of a p-adic group $G(F)$, one can produce a representation of $G(F)$ in two different ways: (twisted) Yu's construction and positive-depth Deligne–Lusztig induction. It is natural to ask when these produce the same representation. In 2023, Chan and Oi proved that if the residue field is sufficiently large, positive-depth Deligne–Lusztig induction and twisted Yu's construction coincide.

In this talk, we will discuss how to extend this result to small residue fields using Shintani lifting of the associated parahoric representations. This gives us a correspondence between these representations of $G(F)$ and those of $G(F')$ coming from the pair $(S(F'), \phi \circ \mathrm{Nm}_{F'/F})$, where F'/F is a finite unramified extension.

SIDDHARTH MAHENDRAKER, Boston College
Regularizing the Geometric RTF for the Galois Period on SL_2

The relative trace formula plays a fundamental role in understanding distinguished automorphic representations. In this talk, I introduce a new regularized relative trace formula for the Galois period on SL_2. I will give an overview of the proof of convergence of the geometric side, and then discuss the fine geometric expansion, with a special focus on the contribution from relative unipotent orbital integrals. It turns out that these can be understood geometrically in terms of the Springer resolution of the nilpotent cone for $\mathrm{Lie}(\mathrm{SL}_2)$. This clarifies and generalizes formulas first written down by Labesse and Langlands in the setting of the usual trace formula.

GILL MOSS, University of Maine
Converse theorems and the local Langlands conjecture in families for GSp(4)

We will outline ongoing joint work-in-progress with Johannes Girsch regarding a generalization of the construction of Novodvorsky local gamma factors for generic representations of GSp(4,F), where F is a nonarchimedean local field of residue characteristic p. More specifically, we generalize the construction to certain classes of representations where the coefficients are a commutative Noetherian ring in which p is invertible. The gamma factors give descent data for families of Whittaker models. We present a heuristic of how this can be applied in the framework of the local Langlands conjecture in families.

ATONU MUKHERJEE, University of British Columbia
Log Canonical Threshold of Vinberg (enveloping) monoid

The Vinberg enveloping monoid is an important example of a spherical variety, and by the Luna–Vust classification, its geometry is encoded by an associated colored cone. This colored cone governs $(G \times G)$-equivariant embeddings and encodes boundary divisors, valuation data, and colors of the spherical variety.

The log canonical threshold (LCT) is a numerical invariant that measures the severity of singularities of a divisor relative to a fixed ambient log structure and plays a central role in the minimal model program.

In this setting, the LCT of the Vinberg enveloping monoid is studied via its colored cone. The main result relates the LCT in terms of root-theoretic and combinatorial data extracted from the colored cone of the Vinberg enveloping monoid.

TAEUK NAM, Harvard University

Categorical Trace and Geometric Langlands

I will speak about the categorical trace construction and how it can be used to traverse between categorical levels. Then, I will discuss some applications of categorical trace to representation theory and automorphic functions, including the connection between geometric Langlands and function-theoretic Langlands via taking categorical trace of Frobenius.

MONICA NEVINS, University of Ottawa

On 4-packets of representations of SL(2,F), for F a 2-adic field

The restriction of an irreducible representation of a reductive p-adic group to a sufficiently small open subgroup is governed by the geometry of nilpotent orbits in the dual of the Lie algebra, in a way made precise for local fields of characteristic zero by the analytic Harish-Chandra–Howe local character expansion. Henniart and Vignéras establish a representation-theoretic statement of this result for SL(2,F) that holds without restriction on the characteristic of F, nor on the ring over which we realize the representations. In prior work, we have proven a variant of this result over $\mathbb{C}$ for $p \neq 2$ and, in joint work with Zander Karaganis, for the case of depth-zero supercuspidal representations when $p = 2$.

In ongoing joint work with David Schwein, we connect the two approaches when $p = 2$ for wildly ramified irreducible supercuspidal representations of SL(2,F), where the key objects are the fascinating collections of representations whose L-packets have four elements. In this talk, we present our recent progress, focussing particularly on the case of 4-packet representations of $\mathrm{SL}(2, \mathbb{Q}_2)$.

NICK ROSENBLYUM, University of Toronto

Categorical representations and traces

I will give a brief overview of the theory of categorical representations and traces. The operation of categorical trace allows to pass from categorical to classical representations. I will describe some applications of these constructions to questions in (usual) representation theory.

THOMAS RÜD, MIT

Jacquet–Rallis Transfer for ramified unitary groups and the Arithmetic Transfer Conjecture

One aspect of relative Langlands program is, given a group G and spherical subgroup H, detecting whether a representation $\pi \subset L^2(G(\mathbb{A}))$ "comes from" a representation of H by a conjectural transfer. This detection relies on the vanishing of period integrals, which themselves are related to the vanishing of automorphic L-functions. For unitary groups, such periods can also be expressed as a generalization of heights of some points in a Shimura variety.

The linear case of Rankin–Selberg Flicker–Rallis periods is historically well-studied, and the modern approach on unitary groups suggested by Gan–Gross–Prasad is to reduce to these two cases by means of the relative trace formula.

The problem is solved for unramified local unitary groups but remains open even in low-dimensional ramified local fields. In this talk I will present what is known and how this relates to orbital integrals and the Arithmetic Transfer Conjecture for Shimura varieties. I will then formulate a conjecture establishing an explicit transfer in the ramified setting, together with a proof for rank one groups. This is joint work with Wei Zhang.