Combinatorial Design Theory
Théorie des plans combinatoires
(Org: **Masoomeh Akbari** (University of Ottawa), **Kianoosh Shokri** (University of Ottawa) and/et **Brett Stevens** (Carleton University))

MASOOMEH AKBARI, University of Ottawa
On the generalized honeymoon Oberwolfach problem

The Honeymoon Oberwolfach Problem (HOP), introduced by Šajna, is a recent variant of the classic Oberwolfach Problem. This problem asks whether it is possible to seat $2m_1 + 2m_2 + \cdots + 2m_t = 2n$ participants, consisting of n newlywed couples, at t round tables of sizes $2m_1, 2m_2, \ldots, 2m_t$ for $2n - 2$ nights, so that each participant sits next to their spouse every night and next to every other participant exactly once. This problem is denoted by $\mathrm{HOP}(2m_1, 2m_2, \ldots, 2m_t)$. Jerade, Lepine, and Šajna have studied the HOP and resolved several important cases.

We generalized the HOP by allowing tables of size two, relaxing the previous restriction that tables must have a minimum size of four. In the generalized HOP, we aim to seat the $2n$ participants at s tables of size 2 and t round tables of sizes $2m_1, 2m_2, \ldots, 2m_t$, where $2n = 2s + 2m_1 + 2m_2 + \cdots + 2m_t$ and $m_i \geq 2$, while preserving the adjacency conditions of the HOP. We denote this problem by $\mathrm{HOP}(2^{\langle s \rangle}, 2m_1, \ldots, 2m_t)$.

In this talk, we will present a general approach to this problem, as well as recent solutions to several cases.

TIM ALDERSON, University of New Brunswick, Saint John
Length-Maximal Nonlinear Codes with Given Singleton Defect–Structure and Bounds

For a linear $[n, k, d]_q$ code, the columns of a generator matrix form a projective arc, and the maximum length is governed by the classical maximal-arc bound $n \leq (s + 1)(q + 1) + k - 2$, where s is the Singleton defect. We show that this same bound holds for all $(n, q^k, d)_q$ codes, with no assumption of linearity. Codes attaining the bound, which we call length-maximal, are necessarily symbol-uniform and have a sharply constrained distance spectrum. They also satisfy a divisibility condition on s that mirrors, but is weaker than, the condition forced on linear codes. An equivalent form of the bound yields an improved Singleton-type inequality that recovers and extends a result of Guerrini, Meneghetti, and Sala for binary systematic codes. When the defect is large, the bound tightens in discrete steps. We also identify several conditions under which nonlinear codes satisfy the Griesmer bound, and close with open problems, grounded by the central question: *can genuinely nonlinear length-maximal codes exist for parameters where no linear codes do*?

SIMON BLACKBURN, Royal Holloway University of London
Tredoku Patterns

Tredoku patterns are certain configurations of diamond-shaped tiles (more precisely, lozenges), inspired by a sukodu-like puzzle that appears in some newspapers such as The Times in the UK. (Take a moment to search the web for tredoku: these patterns can be very pretty!) They were were introduced in two talks by Donald Preece in 2013, in which he conjectured the range of achievable parameters that tredoku patterns can take. This talk will sketch a proof of Donald's conjectures, taken from the following paper:

Simon R. Blackburn, 'Tredoku patterns', https://arxiv.org/html/2407.10752v3.

JOY COOPER, University of Victoria
A Fuzzy Generalization of Latin Squares

In *Six Permutation Patterns Force Quasirandomness* by Crudele, Dukes, and Noel (2024), the authors introduce the notion of a fuzzy permutation matrix and what we now call a fuzzy Latin square. A fuzzy permutation matrix is a way of combinatorially

embedding a permutation matrix of order k in a matrix of order $n \geq k$. We say a Latin square is partially reduced if its first row is $1, 2, \ldots n$. In the same way that a partially reduced Latin square can be expressed as a sum of permutation matrices, a fuzzy Latin square is a linear combination of fuzzy permutation matrices which total to a constant matrix.

Expanding on their work, we bound the dimension of partially reduced Latin squares of order n as a vector space using the representation theory of the symmetric group, investigate vanishing fuzzy Latin squares, and compute the fuzzy Latin squares with six terms.

This work has been supervised by Peter Dukes.

PETER DANZIGER, Toronto Metropolitan University
A complete solution to the directed Oberwolfach problem of order $2 \bmod 4$ *with cycles of even lengths*

The Oberwolfach problem asks for a 2-factorization of the complete graph in which each 2-factor is isomorphic to a specific factor F. Recently, this problem has been extended to directed graphs. In this case, the directed Oberwolfach problem asks for a directed 2-factorization of the complete symmetric digraph in which each directed 2-factor is isomorphic to a specific directed factor F. We consider the directed Oberwolfach problem with directed 2-factors comprised of cycles of even lengths. We provide a complete solution to the case where F is a bipartite 2-factor and the order of the complete symmetric digraph is congruent to 2 modulo 4.

SHONDA DUECK, University of Winnipeg
The threshold strong dimension of the hypercube

A set W of vertices of a connected graph G is a *strong resolving set* for G if, for every pair of vertices, one of the vertices in the pair lies on a shortest path from the other vertex to some vertex of W. The smallest cardinality of a strong resolving set of vertices of G is the *strong dimension* of G. The strong dimension is a stronger version of the better known metric dimension of a graph. Highly symmetric graphs, such as the incidence graphs of combinatorial designs, tend to have low metric and strong dimension, and much work has been done constructing resolving sets for these graphs using properties of the designs from which they arise. The *threshold strong dimension* of G is the smallest strong dimension among all graphs having G as a spanning subgraph, and it is denoted by $\tau_s(G)$. We present a geometric characterization of $\tau_s(G)$, which expresses $\tau_s(G)$ in terms of the smallest number of paths (each of sufficiently large order) whose strong product admits a certain type of embedding of G. We use this geometric characterization as a tool to bound the threshold strong dimension of the hypercube. This is ongoing joint work with Nadia Benakli, Novi H. Bong, Linda Eroh, Beth Novick, and Ortrud Oellermann.

AARON DWYER, Carleton University
Perfect Codes in the Lee Scheme

The theory of error correcting codes is well explored when we consider the distance between two codewords as the Hamming distance. A well known bound in the Hamming metric and its association scheme is the sphere packing bound. Codes that reach this bound are called perfect. All perfect codes in the Hamming scheme are already categorized. Codes defined over cyclic rings with the Lee distance have been less explored. The sphere packing bound and perfect codes can be defined in the Lee scheme. In this talk I aim to explore the basics of Lee codes, focusing on perfect codes, giving some examples and stating some key theorems.

MARIE ROSE JERADE, University of Ottawa
The Honeymoon Oberwolfach Problem: A Recursive Approach

The *Honeymoon Oberwolfach Problem* (HOP), first introduced by Mateja Šajna, is a variant of the Oberwolfach Problem (OP). The problem asks whether it is possible to seat n couples at ℓ round tables of sizes $2m_1, 2m_2, \ldots, 2m_\ell$ for $2(n-1)$ consecutive nights, such that $m_i \geq 2$ for all $i = 1, \ldots, \ell$, $\sum_{i=1}^{\ell} m_i = 2n$, and each participant must be seated next to their spouse every night, but next to every other participant exactly once. This problem is denoted by $HOP(2m_1, 2m_2, \ldots, 2m_\ell)$.

This seating arrangement problem translates naturally into a graph theoretic problem. It is equivalent to finding a cycle decomposition of the graph $K_{2n} + (2n-3)I$ (that is, the complete graph K_{2n} with $2n-3$ additional copies of a fixed 1-factor I) into 2-factors such that each 2-factor consists of disjoint I-alternating cycles of lengths $2m_1, \ldots, 2m_\ell$. So far, solutions to HOP have been established in various cases, including: all $n \leq 20$; the case when all tables are of the same size; the case when all table sizes are divisible by 8; and the case when n is odd and $OP(m_1, \ldots, m_\ell)$ has a solution.

In this talk, we present a recursive approach to the problem. In particular, we show that if $m_1, \ldots, m_\ell$ are positive even integers, t is an integer satisfying $\sum_{i=1}^{\ell} m_i < t$, and $HOP(2m_1, \ldots, 2m_\ell)$ has a solution, then $HOP(2m_1, \ldots, 2m_\ell, 2t)$ also has a solution. Moreover, we briefly discuss the case when $m_1, \ldots, m_\ell$ are not all even.

This is joint work with Mateja Šajna.

SHUXING LI, University of Delaware

Generalized Additive Bases and Difference Bases For Cartesian Product of Finite Abelian Groups

For a finite group G and a positive integer g, a *g-additive basis* is a subset of G whose pairwise sums cover each element of G at least g times; *g-difference bases* are defined analogously using pairwise differences rather than sums. We investigate such bases in the group G^n, the n-fold Cartesian product of a finite abelian group G. We explicitly construct g-additive and g-difference bases in G^n, and in doing so obtain asymptotically sharp upper bounds on the minimal sizes of such bases as n grows.

This talk is based on joint work with Chi Hoi Yip (Georgia Institute of Technology).

ANDREW NAGARAJAH, Carleton University

A brief tour of completion results for partial designs

For many kinds of combinatorial design (broadly interpreted) which have a notion of 'partial design', there is a non-trivial threshold such that 'sufficiently large' partial designs can always be completed. Sometimes 'sufficiently small' partial designs must also have completions, and sometimes no non-trivial statement of this type is true. Sometimes partial designs can instead be extended by, say, adding points. Here we survey some phenomena of the first type from the worlds of block designs, intersecting families, geometry, and others.

PRANGYA PARIDA, University of Ottawa

One Sequence to Rule them All.

A binary reflected Gray code (BRGC), denoted by $B(n)$, is an ordering of the 2^n binary n-tuples such that any two consecutive tuples differ in exactly one bit. This object can also be generated using a greedy approach (introduced by Williams). It is well-known that the position of the bit that changes at each step in the generation of $B(n)$ is given by the binary ruler sequence (OEIS A001511). This leads to a loopless ($\mathcal{O}(1)$-time) generation of $B(n)$ (by Bitner, Ehrlich, and Reingold).

Mixed-radix reflected and modular Gray codes are natural extensions of the BRGC, where the base of each position is not necessarily binary. Their respective loopless algorithms can be found as Algorithm H and Exercise 77 in Section 7.2.1.1 of TAOCP Vol. 4A by Knuth. In this talk, we discuss that both of these Gray codes built on a mixed-radix base are guided by the same change sequence, which allows them to be generated in parallel. We show that modular Gray codes can be generated using a greedy cyclic increment approach. Furthermore, we present a new family of modular Gray codes starting from any tuple w that can be constructed by using the same greedy approach to generate the next word that is lexicographically greater than or equal to w. We show that although this order is not a suffix of the full modular Gray code, its change sequence is a suffix of the latter.

Joint work with Lucia Moura, Brett Stevens, and Aaron Williams.

DAVID PIKE, Memorial University of Newfoundland

Edge-connectivity of vertex-transitive hypergraphs

A graph or hypergraph is said to be vertex-transitive if its automorphism group acts transitively upon its vertices. A classic theorem of Mader asserts that every connected vertex-transitive graph is maximally edge-connected. We generalise this result to hypergraphs and show that every connected linear uniform vertex-transitive hypergraph is maximally edge-connected. By using combinatorial designs, we also show that if we relax either the linear or uniform conditions in this generalisation, then we can construct examples of vertex-transitive hypergraphs which are not maximally edge-connected. This is joint work with Andrea Burgess and Robert Luther.

SAROBIDY RAZAFIMAHATRATRA, Carleton University

Using cyclic codes to solve an Erdős-Ko-Rado type problem on permutation groups

Given a finite transitive group $G \leq \mathrm{Sym}(\Omega)$, we say that a subset $\mathcal{F} \subset G$ is an intersecting set if any two elements of $\mathcal{F}$ agree on some elements of Ω. If $|\Omega| = pq$, for some odd primes $q < p$, and $G \leq \mathrm{Sym}(\Omega)$ is transitive, then it was conjectured that any intersecting set of G has size at most the order of a point stabilizer. This conjecture was recently disproved using certain cyclic codes in $\mathbb{F}_q^p$. In this talk, I will show that the conjecture is essentially true whenever these cyclic codes in $\mathbb{F}_q^p$ do not exist.

This is based on joint work with Roghayeh Maleki and Angelot Behajaina.

SHAHRIYAR POURAKBAR SAFFAR, Memorial University of Newfoundland

On Uniquely Colourable Biclique Designs

A decomposition of a graph G is a collection of subgraphs of G whose union partitions the edge set of G. An (m_1, m_2)-biclique decomposition $\mathcal{D}$ of a graph G is a decomposition of G where $\mathcal{D}$ consists of complete bipartite graphs with parts of size m_1 and m_2. A decomposition $\mathcal{D}$ of G is said to be k-colourable if we can assign k colours to the vertices of G so that no member of $\mathcal{D}$ is monochromatic. If a decomposition is k-colourable but not $(k-1)$-colourable, it is called k-chromatic. A k-colourable decomposition is uniquely k-colourable if its colouring is unique up to the permutation of colour classes.

The study of colouring decompositions has been explored in various settings, but few results are known for unique colourability even for the most prominent families of decompositions such as balanced incomplete block designs and cycle systems. In 2003, Forbes showed that there exist uniquely 3-colourable Steiner triple systems of order n for all admissible $n \geq 25$. More recently, Darijani and Pike proved the existence of uniquely k-colourable e-star systems (for all $k \geq 2$ and $e \geq 3$) and uniquely 2-colourable P_4 systems. Lastly, Burgess, Pike, and Pourakbar-Saffar constructed a uniquely 2-colourable 4-cycle system of order n for all admissible $n \geq 49$.

In this talk, we mainly focus on the construction of uniquely k-colourable (r, r)-biclique decompositions of complete multipartite graphs for all $r \geq 2$ and $k \geq 2$ except $(r, k) = (2, 2)$. These constructions are our stepping stone to find uniquely k-colourable $2r$-cycle systems.

KIANOOSH SKOKRI, University of Ottawa

On the Chromatic Number of Cayley Tables of Dihedral Groups

A coloring of a Latin square is an assignment of k colors to its cells such that no two cells in the same row, the same column, or containing the same symbol receive the same color. The chromatic number of a Latin square L is the minimum number of colors for which such a coloring exists, and is denoted by $\chi(L)$. The Cayley table of a finite group G of order n is a Latin square denoted by L_G. For any finite group G of order n, it is known that $\chi(L_G) \neq n+1$. Hence, $\chi(L_G) = n$ or $\chi(L_G) \geq n+2$. Recently, it was proven that $\chi(L_G) = n$ if and only if every Sylow 2-subgroup of G is either trivial or non-cyclic. Consequently, finite groups G satisfying $\chi(L_G) = n$ can be completely characterized. In this talk, we show that the dihedral group D_n, for odd n, has a cyclic Sylow 2-subgroup, and therefore $\chi(L_{D_n}) \geq 2n+2$. We then prove that $\chi(L_{D_p}) = 2p+2$ for every prime $p > 3$, showing that these Latin squares attain the coloring bound.

This is joint work with E.S. Mahmoodian.

DOUG STINSON, University of Waterloo

Optimal Equidistant Codes–A Detective Story

An *equidistant code* $E(n, d, m)$ consists of m binary codewords of length n, such that the hamming distance between any two distinct codewords is exactly d. We study the following problem: (1) Given n (the length of the codewords), determine the maximum m such an $E(n, d, m)$ exists for some d. Denote this maximum value of m (which of course is a function of n) by m^*. (2) Determine the maximum d such that an $E(n, d, m^*)$ exists. Denote this maximum value of d (which is also a function of n) by d^*. (3) Give a strong combinatorial characterization (in terms of symmetric BIBDs) of $E(n, d^*, m^*)$.

Such optimal equidistant codes have been studied at least since the 1970's. However, many results have been reproved multiple times, and some early work seems to be not well well known. Also, the most comprehensive solution to the problem seems to be incomplete. In this talk, we review the history of this problem and attempt to give a complete unified treatment of the known results.