Stochastic and Singular PDEs, and Related Fields

(Org: **Damir Kinzebulatov** (Université Laval) and/et **Jie Xiao** (Memorial University))

RALUCA BALAN, University of Ottawa
Moment estimates for solutions of SPDEs with Lévy colored noise

In this talk, I introduce a class of processes that can be used as noise for stochastic partial differential equations (SPDEs). This noise is called the *Lévy colored noise*, and is constructed from a Lévy white noise using the convolution with a suitable spatial kernel. We assume that the Lévy measure of the noise has finite variance. Therefore, the stochastic integral with respect to this noise is constructed similarly to the integral with respect to the spatially-homogeneous Gaussian case considered in Dalang (1999). Using Rosenthal's inequality, we provide an upper bound for the p-th moment of the stochastic integral with respect to this noise, which allows us to identify sufficient conditions for the solution of an SPDE driven by this noise to have higher order moments. We first analyze this question for the linear SPDE (in which the noise enters in an additive way), considering as examples the stochastic heat and wave equations. We present a general theory for a non-linear SPDE with Lipschitz coefficients, and perform a detailed analysis in the case of the heat equation (in dimension $d \geq 1$), and wave equation (in dimension $d \leq 3$). We show that the solution of each of these equations has a finite upper Lyapounov exponent of order $p \geq 2$, and in some cases, is weakly intermittent. In the case of the parabolic/hyperbolic Anderson model, we provide the Poisson chaos expansion of the solution, and compute the second-order upper Lyapounov exponent. This talk is based on joint work with Juan Jiménez.

ILIA BINDER, University of Toronto
Multifractal Analysis of Harmonic Measure.

In the talk, we discuss the fine dimensional properties of planar harmonic measure. More specifically, we will examine various notions of the *multifractal spectra of harmonic measure* and discuss sharp bounds on them. Additionally, we will highlight the relationship of the multifractal spectra with the distortion properties of conformal maps near the boundary. The talk is based on joint work with A. Glücksam.

LINAN CHEN, McGill

YU-TING CHEN, Victoria

WEIYANG LI, Memorial University of Newfoundland
Weak/stable solutions to p-Kirchhoff equation: only-zero or non-existence

In this talk, I would like to figure out some natural conditions for a weak/stable solution of the p-Kirchhoff equation

$$-\left(a + b||\nabla u||_p^p\right)\Delta_p u(x) = f_\alpha(|x|)g(u(x)) \;\; \forall \;\; x \in \mathbb{R}^N$$

to be either only-zero or non-existent.

RAPHAEL MADOU, McGill

NGUYEN NGUYEN, Memorial University of Newfoundland
Surface area measures of α-concave functions and their Minkowski problem

An α-concave function serves as a functional generalization of convex bodies, which are main objects in the field of Convex Geometry. Among the key tools for studying convex bodies is the surface area measure, a measure on the sphere that contains the information of surface area of the convex body. In this talk, we first discuss the extension the concept of surface area measures from convex bodies to those of α-concave functions via a variational formula for their total mass. Having defined the surface area measures of α-concave functions, it is natural to consider the Minkowski problem in this generalized setting. Notably, the main tool employed in addressing the Minkowski problem in our setting is optimal transport, which is not a traditional approach to such problems.

JANOSCH ORTMANN, UQAM and CRM
The coupling method for central moment bounds in exponential last-passage percolation

Last passage percolation (LPP) models, central to the Kardar–Parisi–Zhang (KPZ) universality class, are deeply connected to stochastic partial differential equations through scaling limits. In particular, the fluctuation behaviour of LPP models under appropriate scaling converges to the solution of the KPZ equation. In this way, LPP with exponentially distributed weights provides an exactly solvable discrete model whose asymptotics illuminate the behaviour of SPDEs with rough noise and nonlinear interactions.

I will discuss two variants of the exponential LPP model: the bulk model, where the last-passage environment is given by independent Exp(1)-distributed weights on the integer quadrant and the (two-sided) boundary model, where an additional layer of independent Exp(w)-distributed weights are placed on the horizontal boundary and independent Exp(1-z)-distributed weights on the vertical boundary, for $0 < w, z < 1$. A particular interest for the increment-stationary case ($w = z$) arises from the fact that then the distribution of the LPP increments is invariant under lattice shifts.

I will discuss how the coupling framework can be used to derive optimal-order upper and lower bounds on the central moment for these two variants of exponential LPP. That is, letting v be the LPP end-point we obtain bounds proportional to $\|v\|^{p/2}$ (CLT scaling) when v is close to the axis and to $\|v\|^{p/3}$ (KPZ regime) otherwise. These bounds are also uniform over vertices taking values in these regions.

The talk is based on joint work with Elnur Emrah and Nicos Georgiou.

SCOTT RODNEY, Cape Breton

ZACHARY SELK, Queen's University
Onsager-Machlup under Renormalization

Stochastic quantization is a procedure for constructing measures, typically on spaces of distributions, with a given "probability density function" as the invariant measure of a stochastic PDE. These "densities" typically involve nonlinearities of distributions which necessitates renormalization. The renormalization, and the lack of a Lebesgue measure on infinite dimensional spaces leads to the question of in what sense these rigorously constructed measures have the given "density". The Onsager-Machlup function is one rigorous notion of probability density function on infinite dimensional spaces.

We are interested in the Φ^4_d, and related, measures in dimensions $d \leq 3$ arising from EQFT. In dimension 1, no renormalization is required. In dimension 2, Wick renormalization is sufficient and in dimension 3, the theory of regularity structures or paracontrolled calculus can be used to renormalize.

In an ongoing joint work with Ioannis Gasteratos (TU Berlin) we analyze the Onsager-Machlup function of the Φ^4 and related measures.

SHAHAB SHAABANI, Concordia University
A view from above on $\mathrm{JN}_p(\mathbb{R}^n)$

For a symmetric convex body $K \subset \mathbb{R}^n$ and $1 \le p < \infty$, we define the space $S^p(K)$ to be the tent generalization of $\mathrm{JN}_p(\mathbb{R}^n)$, i.e., the space of all continuous functions f on the upper-half space $\mathbb{R}^{n+1}$ such that

$$\|f\|_{S^p(K)} := \big(\sup_{\mathcal{C}} \sum_{B \in \mathcal{C}} |f_B|^p\big)^{\frac{1}{p}} < \infty,$$

where, in the above, the supremum is taken over all finite disjoint collections of homothetic copies of K. It is then shown that the dual of $S^1_0(K)$, the closure of the space of continuous functions with compact support in $S^1(K)$, consists of all Radon measures on $\mathbb{R}^{n+1}$ with uniformly bounded total variation on cones with base K and vertex in $\mathbb{R}^n$. In addition, a similar scale of spaces is defined in the dyadic setting, and for $1 \le p < \infty$, a complete characterization of their duals is given. We apply our results to study dyadic JN_p spaces.

WEI SUN, Concordia University
Periodic and stationary solutions of distribution-dependent SDEs

We investigate the periodic and stationary solutions of distribution-dependent stochastic differential equations. While generally, the semigroups associated with the equations are nonlinear, we show that the methods of weak convergence and Lyapunov functions can be combined to give efficient criteria for the existence of periodic and stationary solutions. Concrete examples are presented to illustrate the novel criteria. This talk is based on joint work with Ethan Wong.

REIHANEH VAFADAR, Laval

DEPING YE, Memorial

CHENGJUN YUE, Memorial University of Newfoundland
Fractional wave potentials and capacities with some applications

We examine the energy estimates and embedding properties of the fractional wave potential arising from the solution to the Cauchy problem for the time and space fractional partial differential equation. We then establish properties of the fractional wave capacity. Finally, we demonstrate some applications of these results to the wave function theory, image denoising, and signal decomposition.

TONG ZHANG, Memorial University
Fractional type inequalities in fractional Sobolev spaces on homogeneous Carnot groups with applications

We investigate several fractional type inequalities in fractional Sobolev spaces on homogeneous Carnot groups. Initially, we establish the equivalence between two types of fractional Sobolev function spaces. Moreover, we demonstrate that the fractional Sobolev inequality on the Lorentz scale is valid in these spaces when $1 \leqslant p < \frac{Q}{\alpha}$. Regarding these fractional Sobolev functions, we prove a fractional Hardy type inequality as well. Furthermore, we establish a fractional Adams–Moser–Trudinger type inequality, which provides exponential integrability for the fractional Sobolev functions when $p = \frac{Q}{\alpha}$. Additionally, a fractional Poincaré type inequality is derived for $p > \frac{Q}{\alpha}$ and $p = \infty$. Meanwhile, the fractional Morrey inequality holds true for $p > \frac{Q}{\alpha}$ and $p = \infty$. Overall, these results provide a systematic generalization of classical inequalities to fractional Sobolev spaces on homogeneous Carnot groups, covering various ranges of p. As an initial application, we explore the existence of weak solutions to p-sub-Laplacian equation and inequalities with fractional order, establishing several significant results. This is a joint work with Jie Xiao.

WEI ZHENZHEN, Memorial University of Newfoundland

Sharp constants and optimizers for the anisotropic Caffarelli-Kohn-Nirenberg inequalities and related identities

The fundamental Caffarelli-Kohn-Nirenberg (CKN) inequality plays a crucial role in functional analysis, partial differential equations, and geometric measure theory. It was originally studied in the isotropic setting with respect to the Euclidean norm of vectors. When the Euclidean norm is replaced by the Minkowski functional of a convex body (i.e., a compact convex subset of $\mathbb{R}^N$ with nonempty interior), the CKN inequality transforms into its anisotropic counterpart, which remains not fully understood.

In this talk, I will discuss anisotropic CKN identities and their inequalities. In particular, I will present our recent progress on sharp constants, the existence and nonexistence of optimizers for the anisotropic CKN inequalities.