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Chevalley operations on TNN Grassmannians

Suppose $\mathbf{X} = (x_{ij})$ is an $n \times n$ matrix of indeterminates, and consider the real span of products of complementary minors: $\det \mathbf{X}_{P,Q} \det \mathbf{X}_{P^c,Q^c}$, where $P, Q \subseteq \{1, 2, \dots, n\}$ have the same cardinality. Rhoades and Skandera [*Ann. Comb.* 2005] showed that this space has dimension equal to the nth Catalan number and identified a basis given by (TNN) Temperley–Lieb immanants. An important application of their result is a classification of determinantal inequalities that arise as real combinations of such products of minors for TNN $\mathbf{X}$, governed combinatorially by noncrossing partitions.

We introduce *Chevalley operations* on index sets and show several applications. Using the seminal bidiagonal factorization theorem for TNN matrices, Chevalley operations yield an algorithmic framework that provides an alternative classification of the inequalities above, from a new perspective closely related to certain sequences of cluster mutations.

This framework yields a new proof of Lam's log-supermodularity of Plücker coordinates [*Current Develop. Math.* 2014], leading to several notable consequences: (a) each positroid cell in Postnikov's decomposition (2006) of the TNN Grassmannian forms a distributive lattice; (b) log-supermodularity implies numerical positivity in the main theorem of Lam, Postnikov, and Pylyavskyy [*Amer. J. Math.* 2007]; and (c) we obtain an independent proof of the coordinatewise monotonicity of ratios of Schur polynomials, originally established by Khare and Tao [*Amer. J. Math.* 2021], which plays a central role in establishing quantitative estimates for entrywise positivity preservers.