
KATE STANGE, University of Colorado Boulder

Reciprocity obstructions in thin orbits (Apollonian circle packings and continued fractions)

Primitive integral Apollonian circle packings are fractal arrangements of tangent circles with integer curvatures. The curvatures form an orbit of a 'thin group,' a subgroup of an algebraic group having infinite index in its Zariski closure. The curvatures that appear must fall into a restricted class of residues modulo 24. The twenty-year-old local-global conjecture states that every sufficiently large integer in one of these residue classes will appear as a curvature in the packing. We prove that this conjecture is false for many packings, by proving that certain quadratic and quartic families are missed. The new obstructions are a property of the thin Apollonian group (and not its Zariski closure), and are a result of quadratic and quartic reciprocity, reminiscent of a Brauer-Manin obstruction. Based on computational evidence, we formulate a new conjecture. This is joint work with Summer Haag, Clyde Kertzer, and James Rickards. Time permitting, I will discuss some results, joint with Rickards, that extend these phenomena to certain settings in the study of continued fractions.