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A century of entrywise positivity preservers: from classical foundations to finite field solutions

Entrywise transforms preserving positive semidefiniteness has a rich history spanning over a century. In 1911, Schur established that the entrywise product of positive semidefinite matrices remains positive semidefinite – a foundational result in matrix analysis. Building on this, Pólya and Szegő in 1925 observed that functions given by power series with nonnegative Maclaurin coefficients, now known as absolutely monotonic functions, preserve positivity when applied entrywise to matrices of all sizes. They also posed a fundamental question: can a non-absolutely monotonic function exhibit the same positivity preserving property?

A major breakthrough came in 1942 when Schoenberg proved that all continuous preservers must indeed be absolutely monotonic. In 1959, Rudin removed the continuity requirement and conjectured a complex analogue: the preservers on complex Hermitian matrices must be power series in z and $\overline{z}$ with nonnegative coefficients; ultimately resolved by Herz in 1963.

These major developments have since inspired a broader theory connecting matrix analysis with fields like metric geometry, high-dimensional statistics, combinatorics, and real/complex analysis. In recent years, significant progress is made to address a challenging refinement of Schoenberg's theorem to characterize positivity preservers on matrices of a fixed dimension, whose partial resolutions involve symmetric function theory and combinatorics.

In this talk, we will survey the evolution of this theory over the past century, culminating in recent results of the speaker with D. Guillot, H. Gupta, and C. H. Yip resolving the aforementioned refinement in the algebraic framework of finite fields, thereby opening new avenues in discrete settings.