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Schwarz Lemma and Multiplier Algebras on Complete Nevanlinna–Pick Spaces

Let $\mathbb{D}$ denote the open unit disk in the complex plane. A classical result in complex analysis, the Schwarz lemma, asserts that if $f : \mathbb{D} \to \mathbb{D}$ is holomorphic with $f(0) = 0$, then f can be written as

$$f(z) = z\, g(z),$$

where g is a holomorphic self-map of $\mathbb{D}$. The collection of all such self-maps forms the unit ball of the algebra of bounded holomorphic functions on $\mathbb{D}$, which is precisely the unit ball of the multiplier algebra of the Hardy space $H^2(\mathbb{D})$. This space has the Szegő kernel as its reproducing kernel:

$$k(z, w) = \frac{1}{1 - z\overline{w}}.$$

Thus, the Schwarz lemma is understood on the unit ball of the multiplier algebra of the Hardy space.

In this talk, we explore a generalization of this perspective by replacing the Szegő kernel with other reproducing kernels—either on $\mathbb{D}$ or on the unit ball in $\mathbb{C}^n$—that satisfy the *complete Nevanlinna–Pick property*. Specifically, we will discuss a version of the Schwarz lemma for the unit ball of the multiplier algebra associated with such spaces.