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The Factor Width and Factor Width Rank of a Matrix

The "factor width" of a positive semidefinite matrix is the smallest positive integer k for which it can be written as a sum of positive semidefinite matrices that are each non-zero only in a single k -by- k principal submatrix. We explore numerous problems related to factor width, such as how we can bound it in terms of the matrix's eigenvalues, and we briefly describe some recently-discovered applications of this quantity in quantum information theory.

We also explore the "factor-width- k rank" of a matrix, which is the minimum number of rank-1 matrices that can be used in a matrix's factor-width-at-most- k decomposition. We show that the factor width rank of a banded or arrowhead matrix equals its usual rank, but for other matrices they can differ. We also establish several bounds on the factor width rank of a matrix, including a tight connection between factor-width- k rank and the k -clique covering number of a graph.