Lie Theory: representations and applications

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YULY BILLIG, Carleton University
Algebraic Gelfand-Fuks cohomology

Starting in the late 1960s, Gelfand's school actively studied cohomology theory for the Lie algebras of vector fields. It was realized, that beyond the simplest examples, computation of the general cohomology of these algebras is intractable. To make computations possible, a special cohomology theory was introduced by Gelfand and Fuks. Many results on Gelfand-Fuks cohomology were established in an analytic setting of C^∞ varieties.

In this talk, we introduce algebraic Gelfand-Fuks cohomology of polynomial vector fields on an affine algebraic variety, with coefficients in differentiable AV-modules. Its complex is given by cochains that are differential operators in the sense of Grothendieck. Using the jets of vector fields, we compute this cohomology for varieties with uniformizing parameters. We prove that in this case, Gelfand-Fuks cohomology with coefficients in a tensor module decomposes as a tensor product of the de Rham cohomology of the variety and the cohomology of the Lie algebra of vector fields on affine space, vanishing at the origin. We explicitly compute this cohomology for affine space, the torus, and Krichever-Novikov algebras.

This talk is based on a joint work with Kathlyn Dykes.

THOMAS BITOUN, University of Calgary
On the D-module of an isolated singularity

Let Z be the germ of a complex hypersurface isolated singularity of equation f. We consider the family of analytic D-modules generated by the powers of 1/f and relate it to the pole order filtration on the top cohomology of the complement of {f=0}.

EMILY CLIFF, Université de Sherbrooke
2-groups and their principal bundles

A 2-group is a categorified version of a group: a category with a multiplication operator, for which all group axioms hold up to natural isomorphism. Similarly, there is a notion of principal bundle for a 2-group. We define the moduli space of flat principal 2-group bundles, and prove that it gives a 2-fibration over the moduli space of flat principal bundles for an ordinary group G. Moreover, when G is finite, this 2-fibration provides a categorification of the Freed–Quinn line bundle, a mapping class group equivariant line bundle arising in Dijkgraaf–Witten theory for the finite group G. This is joint work with Daniel Berwick-Evans, Laura Murray, Apurva Nakade, and Emma Phillips.

NOAH FRIESEN, University of Saskatchewan
Braid group actions, Baxter polynomials, and Yangians

It is well known that the braid group of a simple Lie algebra acts on its integrable representations via exponentials of its Chevalley generators. By modifying this action, we obtain an action of the braid group on a commutative subalgebra of the Yangian by Hopf algebra automorphisms. By dualizing, we recover an action on rational functions constructed in the work of Y. Tan, which was used to obtain a sufficient condition for the cyclicity of any tensor product of irreducible Yangian representations. This talk is based on joint work with Alex Weekes and Curtis Wendlandt.

ARTEM KALMYKOV, McGill University
Shuffle Products, Degenerate Affine Hecke Algebras, and Quantum Toda Lattice

We revisit an identification of the quantum Toda lattice for GL_N and the truncated shifted Yangian of $\mathfrak{sl}_2$, as well as related constructions, from a purely algebraic point of view, bypassing the topological medium of the homology of the affine Grassmannian. For instance, we interpret the Gerasimov-Kharchev-Lebedev-Oblezin homomorphism into the algebra of difference operators via a finite analog of the Miura transform. This algebraic identification is deduced by relating degenerate affine Hecke algebras to the simplest example of a rational Feigin-Odesskii shuffle product. As a bonus, we obtain a presentation of the latter via a mirabolic version of the Kostant-Whittaker reduction.

JOEL KAMNITZER, McGill

Monodromy of eigenvectors of inhomogeneous and trigonometric Gaudin algebras

Gaudin algebras are commutative subalgebras inside tensor products of the universal envelopping algebra of a semisimple Lie algebra. We will study eigenvectors for these algebras acting on tensor product representations. These algebras depend on a choice of parameters and it is interesting to study how the eigenvectors change as we vary these parameters. We will give a combinatorial description of this monodromy using cactus groups and crystals.

ANTUN MILAS, SUNY-Albany

Chiral differential operators and quasi-lisse vertex algebras

Some time ago, Arkhipov and Gaitsgory introduced the algebra of chiral differential operators (CDOs) on a semisimple group G within the framework of chiral algebras. The associated vertex algebra, known as the regular VOA, can be interpreted as a chiral analogue of the classical Peter–Weyl theorem for G. More recently, these vertex algebras have been revisited in the framework of genus zero S-class theories, developed by Arakawa and others. In this talk, we will present a construction of a family of vertex algebra closely related to the algebra of CDOs, but somewhat twisted so that they are conical. We shall discuss their properties in the case of G=SL(2).

CHRISTOPHER RAYMOND, University of Hamburg

Inverse Hamiltonian reduction in representation theory

An important question in vertex operator algebra theory is identifying the appropriate category of modules over a VOA to describe a chiral conformal field theory. The recently developed technique of inverse Hamiltonian reduction builds the simple objects for nonsemisimple and non-finite such categories in a natural way. I will give an introduction to this construction and show how it gives new techniques to study interesting families of weight modules for finite and affine Lie algebras.

HENRIQUE ROCHA, Carleton University

AV-modules

The theory of AV-modules emerged as a framework for advancing the representation theory of Lie algebras of vector fields. Initially studied from an algebraic perspective, these modules have recently been explored through a geometric approach. In this talk, we provide an introduction to the theory of AV-modules, presenting key results and discussing recent developments in the field.

LEONID RYBNIKOV, Universite de Montreal

Bethe suablgebras and wonderful models for toric arrangements

Bethe subalgebras in Yangians are maximal commutative subalgebras responsible for higher integrals of various integrable systems (specifically, XXX Heisenberg chain and its generalizations). We study the natural compactification of the parameter space of quadratic components of Bethe subalgebras in the Yangian of any finite type and show that this compactification is isomorphic to De Concini - Gaiffi projective wonderful model for a root toric arrangement. Conjecturally, this compactification parametrizes all possible degenerations of Bethe subalgebras. We describe explicitly Bethe subalgebras corresponding to boundary points of the compactification. Our main tool is the trigonometric version of the holonomy Lie algebra introduced by Toledano Laredo. This is a joint work with Aleksei Ilin.

YVAN SAINT-AUBIN, Université de Montréal

Properties of the Temperley-Lieb algebra of type B obtained from its Morita-equivalent bound path algebra

The family of Temperley-Lieb algebras TLb_n of type B originates from the description of statistical physics models. Its close ties to the Hecke and the KLR algebras transform its representation theory into a mathematically rich laboratory. Alexis Leroux-Lapierre, Théo Pinet and I used Soergel modules to construct bound quiver algebras that are Morita-equivalent to the blocks of TLb_n. I shall describe some of the properties of these bound path algebras (their center, the existence of a largest projective, their grading) and their consequences for the algebras TLb_n.

HADI SALMASIAN, University of Ottawa

Counting rational points of orbital varieties over finite fields

Let $\mathfrak{g}$ be a semisimple Lie algebra and let $\mathfrak{u}$ be a $\mathfrak{b}$-stable ideal of $\mathfrak{n}$, where $\mathfrak{b} = \mathfrak{h} \oplus \mathfrak{n}$ is a Borel subalgebra of $\mathfrak{g}$. An orbital variety of $\mathfrak{g}$ is the intersection of a nilpotent orbit of $\mathfrak{g}$ with $\mathfrak{u}$. When $\mathfrak{g}$ is of type A, we obtain explicit formulas for the number of $\mathbb{F}_q$-points of an orbital variety, in terms of well-known families of symmetric functions. In the special case where $\mathfrak{u}$ is the nilradical of a parabolic subalgebra, our result specializes to a theorem of Karp and Thomas that provides a formula in terms of coefficients of Macdonald polynomials. We also discuss some applications, e.g., a generalization (with new proof) of the Kirillov-Melnikov-Ekhad-Zeilberger formula for the number of elements of $\mathfrak{n}$ with a given matrix rank. This talk is based on joint work with M. Bardestani, K. Karai, and S. Ram.

CURTIS WENDLANDT, University of Saskatchewan

Graded quantum vertex coalgebras

A vertex algebra can be thought of as a commutative algebra whose multiplication map need not be globally defined and can have certain singularities. Starting from this, the concept of a quantum vertex algebra is obtained by relaxing the requirement that the algebra is commutative and instead imposing only the weaker hypothesis that its product and opposite product are intertwined by a solution of the quantum Yang-Baxter equation. At the turn of the century, Etingof and Kazhdan formalized this concept and constructed a quantum vertex algebra structure on the vacuum module of the Yangian double of the special linear algebra at some fixed level k. This example provides a deformation of the standard level k affine vertex algebra and to this day remains one of the most important and well understood examples of a quantum vertex algebra. In this talk, I will explain how to uniformly generalize this construction starting from the notion of a graded quantum vertex coalgebra, which naturally captures certain key properties exhibited by quantum groups like Yangians. This is based on joint work with Alex Weekes and Matt Rupert.

MALIHE YOUSOFZADEH, University of Isfahan & IPM

Twisted affine Lie superalgebras and finite weight module theory

In this presentation, we talk about finite weight modules over twisted affine Lie superalgebras. We explain how we can characterize the modules using some kind of inductions in both level, critical and nonzero. We then go through the characterization of obtained reduced modules and explain how we can complete the characterization.

KIRILL ZAYNULLIN, University of Ottawa

On the formal Peterson subalgebra and its dual

We discuss a generalization of the Peterson subalgebra to a generalized (oriented) cohomology theory which we call the formal Peterson subalgebra.

One of our results shows that the localized formal Peterson subalgebra for the extended Dynkin diagram of type $\hat{A}_1$ provides an algebraic model for 'quantum generalized cohomology' of the projective line. Hence, confirming and extending the Peterson conjecture for these settings.

We also prove that the dual of the formal Peterson subalgebra (a generalized cohomology of the affine Grassmannian) is the 0th Hochschild homology of the formal affine Demazure algebra. Hence, extending the techniques and results on the Hopf algebroids of structure algebras of moment graphs by [Lanini-Xiong-Z.] to the case of affine root systems.