
Harmonic Analysis: commutative to non-commutative

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BENJAMIN ANDERSON-SACKENAY, University of Victoria

Duality for Amenability of Coideals of Quantum Groups

Coideals are special dynamical objects that are a quantization of homogenous spaces from classical groups to quantum groups. They offer a noncommutative analogue of a subgroup with the caveat that there may exist no underlying quantum group serving as a ‘subgroup’. Just like with groups, and more generally quantum groups, there is a Fourier transform for coideals and hence a corresponding Pontryagin duality. An important thing to be able to do is transfer algebra-analytical properties across this Pontryagin duality. In the more general context of quantum group actions on von Neumann algebras (vNas), a vehicle for transferring properties across Pontryagin duality is the crossed product. Here a result comes in the form of having a certain property of a vNA action of a quantum group \mathbb{G} characterized by the another certain property of the induced action of its dual $\hat{\mathbb{G}}$ on the crossed product of the given vNA. Much progress along these lines, especially for coideals, has been achieved in the past few years by the likes of researchers including (and not limited to) myself, Khosravi, Moakhar, De Ro, and Hataishi. In this talk I will discuss a result along these lines for certain (co)amenability properties of coideals and their so-called (co)duals and, in particular, the involvement of crossed products and their role in duality. I will also present, as an application, the resolution of certain operator algebraic problems regarding the reduced C^* -algebras of quantum groups and boundary theory. I will be discussing a joint work with Fatemeh Khosravi.

JOERI DE RO, Vrije Universiteit Brussel (VUB)

Equivariant Eilenberg-Watts theorem for locally compact quantum groups

Let A and B be two von Neumann algebras. We write $\text{Corr}(A, B)$ for the category of A - B -correspondences, whose objects consist of Hilbert spaces endowed with an appropriate A - B -bimodule structure. As a special case, $\text{Rep}(A) = \text{Corr}(A, \mathbb{C})$ is the category of all normal, unital $*$ -representations of A on Hilbert spaces. In the seventies, M. Rieffel proved that there is a categorical equivalence

$$\text{Corr}(A, B) \simeq \text{Fun}(\text{Rep}(B), \text{Rep}(A)),$$

where the latter is the category of all normal $*$ -functors $\text{Rep}(B) \rightarrow \text{Rep}(A)$. This is a von Neumann algebra version of the celebrated Eilenberg-Watts theorem. In this talk, we explain how this result can be generalized to the setting where the von Neumann algebras A and B are upgraded with actions $A \curvearrowright \mathbb{G}$ and $B \curvearrowright \mathbb{G}$, where \mathbb{G} is a locally compact quantum group.

REZA ESMAILVANDI LERI, Carleton University

Arens regularity and irregularity of ideals in Fourier and group algebras

Let \mathcal{A} be a weakly sequentially complete Banach algebra containing a bounded approximate identity that is an ideal in its second dual \mathcal{A}^{**} . In this talk, we look at Arens regular and strongly Arens irregular closed ideals of \mathcal{A} . We then characterize such ideals in two key examples: the group algebra $L^1(G)$ of a compact group G , and the Fourier algebra $A(\Gamma)$ of a discrete amenable group Γ .

The results in this talk are based on joint work with M. Filali and J. Galindo

MEHDI MONFARED, University of Windsor

On Equidistribution of Continuous Functions Along Monotone Compact Covers

We discuss equidistribution of continuous functions along monotone compact covers as a generalization of equidistribution of sequences. We give a measure theoretic necessary and sufficient condition for equidistribution of continuous functions.

Using almost periodic means, we give an analogue of Weyl's equidistribution criterion for continuous functions taking values in arbitrary topological groups. We discuss an analogue of van der Corput's inequality formulated for vectors in a Hilbert space. We show how the generalized inequality leads to an equidistribution result for functions defined on arithmetic progressions in the lattice N^m , taking values in arbitrary topological groups.

The results in this talk are obtained by my Ph.D. student Y. Zhu and are inspired by earlier works of Weyl, Eckmann, Hlawka and van der Corput.

VOLKER RUNDE, University of Alberta

The indicator function of the anti-diagonal in a locally compact group

We discuss if the indicator function of the anti-diagonal, i.e., $\{(x, x^{-1}) : x \in G\}$ of a locally compact group G can be completely bounded multiplier of the Fourier algebra $A(G \times G)$.

NICO SPRONK, University of Waterloo

Spectra of Beurling algebras

A full description is given of the sets which arise as Gelfand spectra of Beurling algebras with symmetric weights of locally compact abelian groups. This represents joint work with Zhihao Zhang (U. Waterloo).

ROSS STOKKE, University of Winnipeg

Homomorphisms of subalgebras of Fourier–Stieltjes algebras

Let G and H be locally compact groups, and let A be a closed translation-invariant subalgebra of $B(G)$, the Fourier–Stieltjes algebra of G . A homomorphism $\varphi : A \rightarrow B(H)$ is determined by a certain mapping $\alpha : E \subseteq H \rightarrow \Delta(A)$, where $\Delta(A)$ is the Gelfand spectrum of A ; we write $\varphi = j_\alpha$. When A is the Fourier algebra $A(G)$, its spectrum is just G , and in this case many authors, including P. Cohen, M. Ilie, N. Spronk, M. Daws and H.L. Pham, have studied the old problem of characterizing when φ is (completely) positive/contractive/bounded in term of the associated map α . In general, however, $\Delta(A)$ can be quite complicated. I will identify a (often large) Clifford subsemigroup Δ_Z of $\Delta(A)$ and, when α maps into Δ_Z , discuss the relationship between α and $\varphi = j_\alpha$. I will describe $\Delta(A)$ when A is a type of ℓ^1 -direct sum of subalgebras graded over a semilattice, discuss examples, and describe for such A when $\varphi : A \rightarrow B(H)$ is completely positive and completely contractive. In several cases, including when A is any (generalized) spine algebra, $\Delta_Z = \Delta(A)$, and we obtain complete characterizations of these homomorphisms in terms of α . This talk is based on joint work with N. Spronk and A. Thamizhazhagan.

ALEKSA VUJICIC, University of Waterloo

The Spine of Local Fell Groups

Given a locally compact group G , the *spine* of the Fourier-Stieltjes Algebra $A^*(G)$, introduced by M. Ilie and N. Spronk, is a subalgebra of $B(G)$ which contains all $A(H) \circ \eta$ where $\eta : G \rightarrow H$ is a continuous homomorphism. We say a group is *spinal* if $A^*(G)$ is all of $B(G)$. Naturally all compact groups are spinal. A known non-compact example is the Fell group $G = \mathbb{Q}_p \rtimes \mathbb{O}_p^*$, where \mathbb{Q}_p and \mathbb{O}_p are the p -adic numbers and integers respectively. We show that if we replace \mathbb{Q}_p with a totally disconnected local field, then this group is also spinal. To date, these local Fell groups are the only known non-compact spinal groups. We also explore the higher dimensional analogue $G = \mathbb{Q}_p^2 \rtimes \mathbb{O}_p^*$, where we compute the spine explicitly. We show in this case that G is not spinal, though in some sense, it is not much larger than $A^*(G)$.

MATT WIERSMA, University of Winnipeg

On operator Connes-amenability of $B(G)$

Runde introduced Connes-amenability as a notion of amenability for dual Banach algebras in 2001, and subsequently showed that the measure algebra $M(G)$ of a locally compact group is Connes-amenable if and only if G is amenable in 2003. By

analog, one might guess that the Fourier-Stieltjes algebra $B(G)$ is operator Connes-amenable if and only if G is amenable, but this is not the case since it fails for $G = \mathbb{F}_2$ (Runde-Spronk, 2004). In this talk, we will describe conditions that imply the failure of operator Connes-amenable for $B(G)$. This provides the first known examples of groups where $B(G)$ fails to be operator Connes-amenable.

This is based on joint work with V. Runde and N. Spronk.

YONG ZHANG, University of Manitoba

Approximate amenability in type I von Neumann algebras

Must an approximately amenable von Neumann algebra be amenable? The question has been open for at least 20 years. N. Ozawa made the first breakthrough in answering this question in 2004. He showed that $B(\mathcal{H})$ is not approximately amenable for any Hilbert space \mathcal{H} of infinite dimension. Later, V. Runde provided more friendly proof of Ozawa's result in his well-known textbook on amenability.

In this talk, we employ Ozawa-Runde's method to investigate approximate amenability in type I von Neumann algebras. We show that this type of von Neumann algebra is approximately amenable if and only if it is amenable.