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A functorial rectification of finitely cocomplete quasicategories

The study of homotopy theories can be classified into two types: the classical approach of homotopical algebra and the modern one of higher category theory.

Classical models deal with relative categories, categories equipped with a class of weak equivalences. These generalize the notion of homotopy equivalences in the category of topological spaces. This implies that constructions and invariants should be studied up to these weak equivalences instead of isomorphisms. Examples include model categories and cofibration categories.

Modern models rely on the idea that the hom-sets are replaced by spaces of morphisms. This has various implications, such as that compositions of morphisms are defined only up to contractible spaces, or that morphisms should not be compared by equality, but rather by homotopies, themselves subject to comparisons by higher homotopies. Examples include quasicategories and complete Segal spaces.

The former approach is best suited for constructing universal objects whereas the latter approach is used for working with universal properties. Therefore, one is interested in comparing these two approaches. Concretely, we would like to know how classical models translate into modern ones and vice versa.

This talk compares two models of the theory of finitely cocomplete ∞-categories: cofibration categories and finitely cocomplete quasicategories. The equivalence of their theories has originally been proved by Karol Szumiło. We will give an alternative proof that does not rely on a specific choice of a functorial localization and avoids the construction of a quasi-inverse. Instead, we exploit the finite completeness of both homotopy theories.