Combinatorial representation theory
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ESTHER BANAIAN, University of California, Riverside
Web Duality

Two classically studied rings are the homogenous coordinate ring of the Grassmannian of k-planes in n-space and the ring of SL_r tensor invariants. Each of these spaces can be understood through certain planar diagrams called webs. In particular, the rings of SL_3 and SL_4 tensor invariants have rotation-invariant web bases due to Kuperberg and Gaetz–Pechenik–Pfannerer–Striker–Swanson respectively. Fraser, Lam, and Le showed a relationship between these spaces via their immanant map. In some small cases, these three authors observe a phenomenon they call "web duality" in which web immanants, i.e. the output of the immanant map from basis webs, are also web invariants. Moreover, this duality can be realized by taking the transpose of standard young tableaux, which index basis webs. We show that this duality continues to hold for a large set of SL_3 and SL_4 webs. Applications of this duality to cluster variables in the coordinate ring of the Grassmannian will be presented by Kayla Wright in the following talk. This is based on joint work with Catania, Gaetz, Moore, Musiker, and Wright.

GRANT BARKLEY, Harvard University
Torsion classes for affine-type preprojective algebras

Bricks for a preprojective algebra have dimension vectors that are real or imaginary roots. We show that, for affine-type preprojective algebras, the real bricks in a torsion class are determined by the set of their dimension vectors. Furthermore, the sets that arise in this way are exactly the biclosed sets of roots, introduced by Matthew Dyer. This answers a conjecture of Dana, Speyer, and Thomas. We also show how this gives rise to an explicit parametrization of the torsion classes in type $\widetilde{A}$ using translation-invariant total orders, and induces a parametrization of the torsion classes for the completed path algebra of the oriented cycle. The resulting lattice is called the *cyclic Tamari lattice*, which we introduced in joint work with Colin Defant.

AMANDA BURCROFF, Harvard University
Generalized Cluster Algebra Positivity and Applications

Cluster algebras are celebrated for their positivity properties, and this positivity hints at beautiful underlying structure. We study this positivity through both a combinatorial and mirror symmetric lens, giving a new understanding of consistent scattering diagrams in rank 2. We give a combinatorial formula for their wall-function coefficients in terms of new objects on Dyck paths, called tight gradings. We use this manifestly-positive formula to prove Laurent positivity for generalized cluster algebras. Our formula also yields explicit expressions for relative Gromov–Witten invariants on weighted projective planes and the Euler characteristics of moduli spaces of framed stable representations on complete bipartite quivers. This is joint work with Kyungyong Lee and Lang Mou.

JUSTIN DESROCHERS, Université de Sherbrooke
Resolution by Spread Modules over Grid Posets

The representation theory of posets holds great potential for applications in topological data analysis. In traditional persistent homology, a data set is associated with a representation M of a totally ordered poset, and topological information is extracted from the indecomposable summands of M. However, in data sets with varying densities or time dependence, it is natural to consider two-dimensional grid posets.

Higher dimensional grid posets typically have wild representation type, and representations arising from data often exhibit complicated indecomposable summands. Relative projective resolutions offer insights into the structure of a module, without computing a decomposition into indecomposables. These resolutions are analogous to projective resolutions in classical homological algebra.

This presentation introduces the theory of resolutions and approximations, with a focus on spread-resolutions over a grid poset. I provide an upper bound on the spread global dimension of any finite poset. This also gives an upper bound on the spread global dimension of certain infinite posets. Throughout, I will highlight several techniques for bounding the relative global dimension of a finite dimensional algebra.

This presentation is based on recent work by Benjamin Blanchette, Eric Hanson, Luis Scoccola, and me.

BENJAMIN GRANT, University of Connecticut
3n-vertex quivers from knot diagrams

We investigate a class of quivers arising from unoriented knot diagrams which are a modified version of the knot quivers of Bazier-Matte and Schiffler. Unlike the BMS knot quivers, however, these quivers remember over- and under-crossing information from the diagram. This is accomplished by including extra vertices for crossing points in the diagram. In this context, mutation at these extra vertices corresponds to flipping between over- and under-crossings. These quivers naturally arise as the quivers of triangulations of punctured spheres, allowing us to obtain a clear understanding of their cluster algebras.

BLAKE JACKSON, University of Connecticut
A geometric model for the non-τ-rigid modules of type $\widetilde{D}_n$

We give a geometric model for the non-τ-rigid modules over acyclic path algebras of type $\widetilde{D}_n$. Similar models have been provided for module categories over path algebras of types $A_n, D_n,$ and $\widetilde{A}_n$ as well as the τ-rigid modules of type $\widetilde{D}_n$. A major draw of these geometric models is the "intersection-dimension formulas" they often come with. These formulas give an equality between the intersection number of the curves representing the modules in the geometric model and the dimension of the extension spaces between the two modules. Essentially, this formula allows us to calculate the homological data between two modules combinatorially. Since there are infinitely many distinct homogenous stable tubes in the regular component of the Auslander-Reiten quiver of type $\widetilde{D}_n$, all of which are disjoint, our geometric data requires an extra decoration on the admissible tagged edges in our geometric model to prevent intersections between curves corresponding to modules in distinct connected components of the Auslander-Reiten quiver.

SHIPING LIU, Université de Sherbrooke
Representations of hereditary artin algebras of Dynkin type

Let H be a hereditary artin algebra of finite representation type. We shall study the category mod$H$ of finitely generated left H-modules with a connection to the bounded derived category $D^b(\mathrm{mod}H)$ and the associated cluster category C_H. By determining all its hammocks, we provide an effective method to construct the Auslander-Reiten quiver of $\mathrm{mod}H$ by simply viewing the ext-quiver of H. As easy applications, we compute the numbers of non-isomorphic indecomposable objects in $\mathrm{mod}H$ and C_H, and also the nilpotency of the radicals of $\mathrm{mod}H$, $D^b(\mathrm{mod}H)$ and C_H.

SCOTT NEVILLE, University of Michigan
Cyclically ordered quivers

Quivers and their mutations play a fundamental role in the theory of cluster algebras. We focus on the problem of deciding whether two given quivers are mutation equivalent to each other. Our approach is based on introducing an additional structure of a cyclic ordering on the set of vertices of a quiver. This leads to new powerful invariants of quiver mutation.

This talk is partially based on joint work with Sergey Fomin.

CHARLES PAQUETTE, Royal Military College of Canada / Queen's University
Brick directed algebras and brick-splitting torsion classes

In this talk, we will explore a new class of algebras called brick-directed algebras, which can be defined as those algebras having no oriented cycles of bricks in their module category. We characterize these algebras from many different viewpoints, including from their torsion theory and their wall-and-chamber structure. A key feature arising in the study of these algebras is the notion of brick-splitting torsion pairs, which are those torsion pairs with the property that any given brick is either torsion or torsion-free. We completely characterize the brick-splitting torsion classes combinatorially within the lattice of torsion classes and derive some consequences of this. This is joint work with Sota Asai, Osamu Iyama and Kaveh Mousavand.

THÉO PINET, McGill University
Soergel algebras as bound-quiver algebras: the infinite dihedral case

The *blob algebra* TLb_N is a finite-dimensional cellular quotient of the Hecke algebra of type B. It appears naturally in statistical physics and admits a notoriously intricate representation theory.

In this talk, we use *Soergel bimodules of affine type* A_1 to give a *bound-quiver algebra realization* for blocks of TLb_N. We then use this novel realization to deduce surprising results about the endomorphism algebras of indecomposable projective TLb_N-modules and obtain, in particular, generalizations of Soergel's famous *Endomorphismensatz* and *Struktursatz*.

The talk is based on joint work with Alexis Leroux-Lapierre and Yvan Saint-Aubin.

DEEPANSHU PRASAD, Queen's University
Galois coverings and mutation of G-orbits

For an algebraically closed field $\mathbb{K}$, we consider a Galois G-covering $\mathcal{B} \to \mathcal{A}$ between locally bounded $\mathbb{K}$-categories given by bound quivers, where G is torsion-free and acts freely on the objects of $\mathcal{B}$. We define the notion of $(G, \tau_{\mathcal{B}})$-rigid subcategory and of support $(G, \tau_{\mathcal{B}})$-tilting pairs over $\mathcal{B}\text{-mod}$. These are the analogues of the similar concepts in the context of a finite-dimensional algebra, where we additionally require that the subcategory be G-equivariant. When $\mathcal{A}$ is a finite-dimensional algebra, we show that the corresponding push-down functor $\mathcal{F}_\lambda : \mathcal{B}\text{-mod} \to \mathcal{A}\text{-mod}$ sends $(G, \tau_{\mathcal{B}})$-rigid subcategories (respectively support $(G, \tau_{\mathcal{B}})$-tilting pairs) to $\tau_{\mathcal{A}}$-rigid modules (respectively support $\tau_{\mathcal{A}}$-tilting pairs). We further show that there is a notion of mutation for support $(G, \tau_{\mathcal{B}})$-tilting pairs over $\mathcal{B}\text{-mod}$. Mutations of support $\tau_{\mathcal{A}}$-tilting pairs and of support $(G, \tau_{\mathcal{B}})$-tilting pairs commute with the push-down functor. We derive some consequences of this, and in particular, we derive a τ-tilting analogue of the result of P. Gabriel that locally representation-finiteness is preserved under coverings. Finally, we prove that when the Galois group G is finitely generated free, any rigid $\mathcal{A}$-module (and in particular $\tau_{\mathcal{A}}$-rigid $\mathcal{A}$-modules) lies in the essential image of the push-down functor.

GORDANA TODOROV, Northeastern University
Infinite friezes of affine type D

We study infinite friezes arising from cluster categories of affine type D and determine the growth coefficients for these friezes. We prove that for each affine type D, the friezes given by the tubes all have the same growth behavior.

KAYLA WRIGHT, University of Oregon
Twists, Higher Dimers and SL_3 and SL_4 Webs in Grassmannian Cluster Algebras

As a sequel to Esther Banaian's talk, we will deepen and make more concrete the connection between SL_k webs and Grassmannian cluster algebras. We will show that the duality of the web bases presented in Esther's talk tells us information about twists of cluster variables. Namely, dual webs govern combinatorial formulae using higher dimer covers for twists of cluster variables. Moreover, we use the machinery of our dual web bases to classify (up to the widely believed Fomin–Pylyavskyy conjectures) cluster variables of rank 3 and 4 in $Gr(3, n)$ and $Gr(4, n)$.