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On the Annihilator of a Pair of Commuting Contractions

Given a contraction T on a Hilbert space $\mathcal{H}$, the *annihilator* of T is defined as

$$\mathrm{Ann}(T) = \{f \in H^\infty(\mathbb{D}) : f(T) = 0\},$$

which is a weak-$*$ closed ideal in the Banach algebra $H^\infty(\mathbb{D})$ of bounded analytic functions on the open unit disc $\mathbb{D}$. When this ideal is non-trivial, Beurling's theorem asserts that it is generated by an inner function θ. To study such annihilators, one introduces the notion of the *support* of an inner function, defined as the set of points in $\overline{\mathbb{D}}$ where the function either vanishes or cannot be analytically continued through the point. This support encodes both spectral and geometric information about $\mathrm{Ann}(T)$: it coincides with the spectrum of T, and the zero set of θ corresponds precisely to the point spectrum of T.

In this talk, we will discuss the annihilator of a pair of commuting contractions on a Hilbert space, and examine how the notions of support and spectral theory relate in this multivariable setting.

This is ongoing joint work with Prof. Raphaël Clouâtre.