Combinatoire algébrique et énumérative

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ANTOINE ABRAM, Université du Québec à Montréal

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Vine model for double forest polynomials

Together with Lucas Gagnon, Philippe Nadeau, Hunter Spink, and Vasu Tewari, we introduced double forest polynomials in our study of equivariant quasisymmetric functions and their connections to geometry. In this talk, I will discuss the vine model, which provides a framework for computing double forest polynomials.

ELISABETH BULLOCK, Massachusetts Institute of Technology

Ehrhart series of alcoved polytopes

In this talk (based on joint work with Yuhan Jiang), I will describe a general method for computing the Ehrhart series of any alcoved polytope via a particular shelling order of its alcoves. As an application, we get a bijective proof of the formula for the Ehrhart h*-polynomial of the second hypersimplex $\Delta_{2,n}$ in terms of Nick Early's decorated ordered set partitions.

ANGELA CARNEVALE, University of Galway, Ireland

Coloured shuffle compatibility and Hadamard products

In this talk I will present recent work on coloured shuffle compatibility of permutation statistics and its applications to zeta functions in algebra. I will discuss how we extended recent work of Gessel and Zhuang, introducing shuffle algebras associated with coloured permutation statistics. Our shuffle algebras provide a natural framework for studying Hadamard products of certain rational generating functions. As an application, we will see how to explicitly compute such products in the context of so-called class- and orbit-counting zeta functions of direct products of suitable groups. This is joint work with V. D. Moustakas and T. Rossmann.

SERGI ELIZALDE, Dartmouth

A bijection for descent sets of permutations with only even and only odd cycles

It is known that, when n is even, the number of permutations of $\{1, 2, \dots, n\}$ all of whose cycles have odd length equals the number of those all of whose cycles have even length. Adin, Hegedűs and Roichman recently found a surprising refinement of this equality, showing that it still holds when restricting to permutations with a given descent set J on one side, and permutations with ascent set J on the other. Their proof is algebraic and relies on higher Lie characters. It also yields a version for odd n.

In this talk we give a bijective proof of the refined identity. First, using known bijections of Gessel, Reutenauer and others, we restate it in terms of multisets of necklaces, which we interpret as words. Then, we construct a weight-preserving bijection between words all of whose Lyndon factors have odd length and are distinct, and words all of whose Lyndon factors have even length.

ALEJANDRO GALVAN, Dartmouth College

YAN LANCIAULT, Université du Québec à Montréal

GAYEE PARK, Dartmouth College
Naruse hook formula for mobile posets

Linear extensions of posets are important objects in enumerative and algebraic combinatorics that are difficult to count in general. Families of posets like Young diagrams of straight shapes and d-complete posets have hook-length product formulas to count linear extensions, whereas families like Young diagrams of skew shapes have determinant or positive sum formulas like the Naruse hook-length formula from 2014. In 2020, Garver et. al. gave determinant formulas to count linear extensions of a family of posets called mobile posets that refine d-complete posets and border strip skew shapes. We give a Naruse type hook-length formula to count linear extensions of such posets by proving a major index q-analogue. We also give an inversion index q-analogue of the Naruse formula for mobile tree posets.

SASHA PEVZNER, Northeastern University

COLLEEN ROBICHAUX, UCLA
Signed puzzles for Schubert coefficients

We give a signed puzzle rule to compute Schubert coefficients. The rule is based on a careful analysis of a recurrence of Knutson. We use the rule to prove polynomiality of the sums of Schubert coefficients with bounded number of inversions. This is joint work with Igor Pak.

ANDREW SACK, University of Michigan

KARTIK SINGH, University of Waterloo

HUNTER SPINK, University of Toronto

TIANYI YU, UQAM
Normal Crystals for symmetric Grothendieck Polynomials

Schur polynomials form a fundamental basis for symmetric polynomials. Motivated by geometry and representation theory, researchers have expanded various polynomials into the Schur basis positively, including (i) skew Schur polynomials, (ii) products of Schur polynomials, and (iii) Stanley symmetric polynomials. Normal crystals provide an elegant framework that effectively demonstrates these Schur expansions. Symmetric Grothendieck polynomials are non-homogeneous analogues of Schur polynomials, arising from the K-theory of flag varieties. Analogous expansions into symmetric Grothendieck polynomials have garnered significant attention over the past decades: Buch established the K-theoretic analogues of (i) and (ii), while Buch, Kresch, Shimozono, Tamvakis, and Yong resolved (iii). In this talk, we present an analogue of normal crystal theory, introducing a powerful new tool for establishing symmetric Grothendieck positivity. This framework not only recovers the three K-theoretic expansions mentioned above but also sheds light on related problems, including a conjecture of Ikeda and Naruse. This work is based on a joint work with Eric Marberg and Kam Hung Tong.