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A bijection for descent sets of permutations with only even and only odd cycles

It is known that, when n is even, the number of permutations of $\{1, 2, \ldots, n\}$ all of whose cycles have odd length equals the number of those all of whose cycles have even length. Adin, Hegedűs and Roichman recently found a surprising refinement of this equality, showing that it still holds when restricting to permutations with a given descent set J on one side, and permutations with ascent set J on the other. Their proof is algebraic and relies on higher Lie characters. It also yields a version for odd n.

In this talk we give a bijective proof of the refined identity. First, using known bijections of Gessel, Reutenauer and others, we restate it in terms of multisets of necklaces, which we interpret as words. Then, we construct a weight-preserving bijection between words all of whose Lyndon factors have odd length and are distinct, and words all of whose Lyndon factors have even length.