SPENCER BACKMAN, University of Vermont

Higher-Categorical Associahedra

In 2017, Bottman introduced a family of posets called 2-associahedra as a tool for investigating functoriality properties of Fukaya categories, and he conjectured that they could be realized as face posets of convex polytopes. We introduce a family of posets called categorical n-associahedra, which naturally extend Bottman's 2-associahedra and the classical associahedra. Categorical n-associahedra give a combinatorial model for the poset of strata of a compactified real moduli space of a tree arrangement of affine coordinate subspaces. We construct a family of complete polyhedral fans, called velocity fans, whose coordinates encode the relative velocities of pairs of colliding coordinate subspaces, and whose face posets are the categorical n-associahedra. In particular, this gives the first fan realization of 2-associahedra. In the case of the classical associahedron, the velocity fan specializes to the normal fan of Loday's realization of the associahedron. Time permitting, we will discuss current investigations of projectivity of velocity fans. This is joint work with Nathaniel Bottman and Daria Poliakova.