Analysis and probability, and their interactions
(Org: **Ilia Binder** and/et **William Verreault** (University of Toronto))

ALMUT BURCHARD, University of Toronto
A max-min property of the ball

I will present recent joint work with Davide Carazzato and Ihsan Topaloglu on maximizing a functional that involves the minimization of the Wasserstein distance between disjoint sets of equal volume. The functional appears as a repulsive interaction term in some models describing biological membranes. Using a symmetrization-by-reflection technique, we show that balls are the unique maximizers of the functional.

LINAN CHEN, McGill University
Exceptional sets of Gaussian Free Fields in Dimension d>2

Originated from quantum mechanics and statistical physics, the log-correlated Gaussian free field (GFF) in 2D has been extensively studied in recent years. In particular, as a natural model of "random surface", the study of GFF has lead to rich developments in the realm of 2D random geometry. However, much less is known in higher dimensions. As an attempt to explore higher-dimensional random geometry, we turn to a class of GFF models in dimensions d>2. In this talk, we will introduce GFFs in any dimension from the viewpoint of abstract Wiener space and discuss some problems arising from the study of the geometry of GFFs, such as the exceptional sets and the multifractal property. In dim d>2, the GFFs will be polynomially correlated and hence more singular compared with the 2D log-correlated model. We will explain the techniques on how to handle the GFFs with worse singularity, and to extend some of the results from 2D random geometry to higher dimensions.

KRZYSZTOF CIOSMAK, University of Toronto
Characterisation of optimal solutions to second-order Beckmann problem through bimartingale couplings and leaf decompositions

The aim of the talk is to present the three-marginal optimal transport problem - introduced in [K. Bolbotowski, G. Bouchitté, Kantorovich-Rubinstein duality theory for the Hessian, 2024, preprint], whose relaxation is the second-order Beckmann problem, and to give a complete characterisation of the optimal solutions for arbitrary pairs $\mu,\nu \in \mathcal{P}(\mathbb{R}^n)$ of absolutely continuous measures with common barycentre such that there exists an optimal plan with absolutely continuous third marginal.

In the talk, I will define the concept of bimartingale coupling for a pair of measures and establish several equivalent conditions that ensure such couplings exist. One of these conditions is that the pair is ordered according to the convex-concave order, thereby generalising the classical Strassen theorem. Another equivalent condition is that the dual problem associated with the second-order Beckmann problem attains its optimum at a $\mathcal{C}^{1,1}(\mathbb{R}^n)$ function with isometric derivative.

I will prove that the problem for the pair μ,ν completely decomposes into a collection of simpler problems on the leaves of the 1-Lipschitz derivative Du of an optimal solution $u \in \mathcal{C}^{1,1}(\mathbb{R}^n)$ for the dual problem. On each such leaf the solution is expressed in terms of bimartingale couplings between the conditional measures of μ,ν, where the conditioning is defined relative to the foliation induced by Du.

DMITRY JAKOBSON, McGill university
Fractional colouring and graph limits

We discuss fractional colouring of graphs and define a new graph invariant that looks like a fractional colouring analogue of entropy per vertex. We study this invariant and discuss several related questions about graph limits. This is joint work with P. Cusson, M. Glasman, G. Lantagne, K. H. Xiao, and C. Zhao

DAMIR KINZEBULATOV, Université Laval

Many-particle Hardy inequality and singular diffusions

I will talk about an improved upper bound on the best possible constant in the many-particle Hardy inequality. Specifically, our result improves the previously known factorial dependence on the dimension to a polynomial dependence. The bound is obtained as an application of our recent work on singular stochastic differential equations, along with classical results on Bessel processes.

To our knowledge, this is the first instance where a probabilistic argument is employed in the analysis of the constant in a Hardy inequality. Our estimate also shows that the lower bound established earlier by Hoffmann-Ostenhof, Hoffmann-Ostenhof, Laptev, and Tidblom is in fact close to optimal.

Joint work with Reihaneh Vafadar.

MICHAEL KOZDRON, University of Regina

Positive Operator Valued Measures and a Quantum Bayes' Rule

A positive operator valued probability measure (POVPM) is a function on a sigma-algebra of subsets of a (locally compact and Hausdorff) sample space that satisfies the formal requirements for a probability, but where its values are positive operators acting on a complex Hilbert space, and a quantum random variable is a measurable operator valued function. Although quantum probability measures and random variables are used extensively in quantum mechanics, some of the fundamental probabilistic features of these structures remain to be determined. In this talk we discuss quantum analogues of the expected value of a random variable and the Radon-Nikodym derivative. This enables us to develop quantum analogues of Bayes' rule and a martingale convergence theorem. This research is based on joint work with Doug Farenick, Sarah Plosker, Kyler Johnson, and Mahbuba Rahman.

ELLIOT PAQUETTE, McGill University

From random matrices, through magic squares, to the multiplicative chaos

In 2004, motivated by connections of random matrix theory to number theory, Diaconis and Gamburd showed a fascinating connection between the enumeration problem of magic squares (squares filled integers with row and column sum constraints) and the moments of the 'secular coefficients' of random matrices, when the size of the matrix tends to infinity. These are the coefficients in the monomial expansion of a characteristic polynomial, or equivalently, the elementary symmetric polynomials of the eigenvalues of this random matrix.

It turns out that this characteristic polynomial has a limit, when the matrix size tends to infinity. It converges to a random fractal, the holomorphic multiplicative chaos. We describe this process on the unit circle, and show how it can be connected even more strongly to random matrices, and how magic square combinatorics are a type of 'signature' of this holomorphic multiplicative chaos. We'll review some open questions about these objects, and discuss some links between this and other stochastic processes such as the Gaussian multiplicative chaos, the 'circular beta-ensemble' and random multiplicative function.

PIERRE-OLIVIER PARISÉ, Université du Québec à Trois-Rivières

The Matching Problem in the Complex Plane

The *matching problem* for a Jordan curve was first introduced by Ebenfelt, Khavinson, and Shapiro in the context of solving the Dirichlet problem using a double-layer potential. They demonstrated that the problem admits a solution when Γ is a rational lemniscate, but has no solution when Γ is the image of the unit circle under a rational map holomorphic in a neighborhood of the closed unit disk. Since then, little progress has been made on the matching problem.

In this talk, I will present a connection between the matching problem for a Jordan curve and the conformal welding of the curve. Leveraging this relationship, I will establish the existence of solutions to the matching problem for certain fractal-like curves. This answers a question posed by the original authors regarding the interplay between the regularity of the curve and the existence of solutions to the matching problem.

Joint work with Kirill Lazebnik and Malik Younsi.

JULIAN RANSFORD, University of Cambridge
ℓ^2 distortion of random planar maps

How well can a planar graph be embedded in a Hilbert space? A theorem of Rao states that every planar graph with n vertices can be embedded in ℓ^2 in such a way that distances do not get distorted by more than a factor of $C\sqrt{\log n}$, where C is some universal constant. Rao's bound is known to be sharp, however the graphs that achieve it are pathological and fractal-like, and so it is natural to ask what happens for a typical planar graph. I will discuss an ongoing project in which we show that random triangulations have ℓ^2 distortion of at least $C\sqrt[4]{\log n}$ with high probability. This is joint work with Jason Miller.