New frontiers for delay models and fractional differential equations

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Nonlinear effects in linear delay equations and linear approach to nonlinear models with delay mortality

In the first part of the talk, we consider a scalar linear mixed differential equation with several terms, both delayed and advanced arguments and a bounded right-hand side. Assuming that the deviations of the argument are bounded, we present sufficient conditions when there exists a unique bounded solution on the positive half-line. Explicit tests are obtained when a bounded solution of a homogeneous equation decays exponentially. Existence of exponentially decaying solutions for this class of differential equations has not been studied before. We show that the standard approach when convergence of all solutions is stated does not work for mixed equations; in addition to an exponentially decaying, there may be a growing solution. All the coefficients and the mixed arguments are assumed to be Lebesgue measurable functions, not necessarily continuous. Though the equation is linear, some properties, as well as the methods applied, are more typical for nonlinear models, for example, fixed-point theorems used in the proofs.

In the second part, we explore existence of positive solutions, persistence, and boundedness of solutions for the Nicholson blowflies model with delayed mortality term $-\delta N(h(t))$. Two global stability tests for the positive equilibrium are obtained based on a linearized global stability method, reducing stability of a non-linear model to a specially constructed linear equation. The first test extends the absolute stability result to the case of delayed mortality, and the second one is delay-dependent.

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Modeling the dynamics of user adoption and abandonment for a single product

We present a compartmental differential equation model to explore the dynamics of user adoption and abandonment for a single product. Our model incorporates two distinct types of abandonment: infectious abandonment, driven by interactions among current and former users, and non-infectious abandonment, triggered by factors such as mass media, advertisements, or the introduction of new products. Unlike previous studies, we treat the infectious abandonment coefficient as a variable that changes linearly with the number of previous users, rather than as a constant. This introduces additional complexity to the model while also enriching its dynamical behavior. We investigate the existence of equilibria of the model and derive the threshold quantity $\mathcal{R}_0$. The user-free equilibrium is always present, and its stability is analyzed under the condition $\mathcal{R}_0 < 1$. Furthermore, we show that a user-prevailing equilibrium does not exist when $\mathcal{R}_0 \leq 1$, but at least one user-prevailing equilibrium is guaranteed when $\mathcal{R}_0 > 1$. We determine the criteria for the existence of one, two, or three user-prevailing equilibria and establish the conditions under which S-shaped and saddle-node bifurcations can arise. Additionally, we establish criteria for different types of Hopf bifurcations. We explore an optimal control problem related to the model, identifying the system that must be satisfied by the optimal control pair. Our theoretical results are validated through extensive numerical simulations.

To demonstrate the practical applicability of our model, we calibrate it using historical data on LinkedIn registered users. The calibrated model is employed to provide forecasts for future user adoption trends.

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A generalized time-fractional convection problem with variable coefficients

Applying the inverse operator method and the multivariate Mittag-Leffler function, we derive a unique analytic solution to the following multi-term time-fractional convection problem on a new space with variable coefficients and $0 < \rho_1 < \rho_2 \cdots < \rho_m < \rho \leq 1$, for the first time, in the Caputo fractional derivative sense:

$$\begin{cases} \frac{{}_c\partial^{\rho}}{\partial t^{\rho}} M(t,\sigma) + \sum_{i=1}^{m} \beta_i \, \frac{{}_c\partial^{\rho_j}}{\partial t^{\rho_j}} M(t,\sigma) + \sum_{j=1}^{n} \lambda_j(\sigma_j) \frac{\partial}{\partial \sigma_j} M(t,\sigma) \\ = f_1(t,\sigma), \quad (t,\sigma) \in [0,1] \times [0,1]^n, \\ M(0,\sigma) = f_2(\sigma). \end{cases}$$

We further present several examples demonstrating power and simplicity of our main results and show that they can be reduced to the classical integral convolution solutions by Green's functions.

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