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Modeling the dynamics of user adoption and abandonment for a single product

We present a compartmental differential equation model to explore the dynamics of user adoption and abandonment for a single product. Our model incorporates two distinct types of abandonment: infectious abandonment, driven by interactions among current and former users, and non-infectious abandonment, triggered by factors such as mass media, advertisements, or the introduction of new products. Unlike previous studies, we treat the infectious abandonment coefficient as a variable that changes linearly with the number of previous users, rather than as a constant. This introduces additional complexity to the model while also enriching its dynamical behavior. We investigate the existence of equilibria of the model and derive the threshold quantity $\mathcal{R}_0$. The user-free equilibrium is always present, and its stability is analyzed under the condition $\mathcal{R}_0 < 1$. Furthermore, we show that a user-prevailing equilibrium does not exist when $\mathcal{R}_0 \leq 1$, but at least one user-prevailing equilibrium is guaranteed when $\mathcal{R}_0 > 1$. We determine the criteria for the existence of one, two, or three user-prevailing equilibria and establish the conditions under which S-shaped and saddle-node bifurcations can arise. Additionally, we establish criteria for different types of Hopf bifurcations. We explore an optimal control problem related to the model, identifying the system that must be satisfied by the optimal control pair. Our theoretical results are validated through extensive numerical simulations. To demonstrate the practical applicability of our model, we calibrate it using historical data on LinkedIn registered users. The calibrated model is employed to provide forecasts for future user adoption trends.