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A search for integrable evolution equations with Lax pairs over the octonions

This talk reports on work searching for octonion evolution equations of KdV type and mKdV type that have a Lax pair.

We consider $u(t,x)$ as an octonion variable in evolution equations $u_t = F(u, u_x, u_{xx}, u_{xxx})$, and we aim to find a Lax pair $L_t = [M, L]$ where L and M are linear differential operators in terms of ∂_x with coefficients involving u and x-derivatives of u. For F, we assume it is homogeneous under a scaling of t, x, u which is either the scaling in the KdV equation or the mKdV equation. This gives a polynomial ansatz with undetermined (real) coefficients. Similarly, for L and M, we assume they are scaling homogeneous, where the scaling weight of M is the same as that of ∂_t while the scaling weight of L can be chosen freely.

The determining condition is $\big(L_t - [M, L]\big)\big|_{u_t=F} = 0$. We split this condition in the jet space of u, and do a further splitting with respect to a real basis (8-dimensional) for the octonions. This gives a large overdetermined system in the undetermined (real) coefficients in ansatz for F, L, M. We use Maple to do the splittings, and depending on the complexity of the system, we solve it using 'rifsimp' in Maple or a package called 'Crack' in Reduce.

As a main result, we obtain a single KdV octonion equation, three mKdV octonion equations, and also a single potential-KdV octonion equation, each of which has more than one Lax pair.