Number theory by early career researchers
Théorie des nombres par les chercheurs en début de carrière
(Org: **Félix Baril Boudreau** (University of Lethbridge) and/et **Nicolo Fellini** (Queen's University))

ABHISHEK BHARADWAJ, Queen's University
On a conjecture of Erdős

In a written communication to Livingston, Paul Erdős proposed the following conjecture:

If N is a positive integer and f is an arithmetic function with period N and $f(n) \in \{-1, 1\}$ when $n = 1, 2, \ldots, N-1$ and $f(n) = 0$ whenever $n \equiv 0 \bmod N$ then $\sum_{n \geq 1} \frac{f(n)}{n} \neq 0$.

We describe the literature around this conjecture, and mention some new results. This is an ongoing joint work with Ram Murty and Siddhi Pathak.

JÉRÉMY CHAMPAGNE, University of Waterloo
Weyl's equidistribution theorem in function fields

Finding a proper function field analogue to Weyl's theorem on the equidistribution of polynomial sequences is a problem that was originally considered by Carlitz in 1952. As noted by Carlitz, Weyl's classical differencing methods can only handle polynomials with degree less than the characteristic of the field. In this talk, we discuss some recent methods which avoid this "characteristic barrier", and we show the existence of polynomials with extremal equidstributive behaviour.

This is joint work with Yu-Ru Liu, Thái Hoàng Lê and Trevor D. Wooley.

NIC FELLINI, Queen's University
Congruence relations for class numbers of real quadratic fields

In 1951, Ankeny, Artin, and Chowla released a short note containing four congruence relations involving the arithmetic invariants of $\mathbb{Q}(\sqrt{d})$ for $d = 1 \bmod 4$. They proved three of these relations the following year, in a paper published in the Annals of Mathematics. Their proof uses a combination of p-adic and group ring theoretic methods. In this talk I will indicate how p-adic L-functions can be used to obtain congruence relations involving the arithmetic invariants of $\mathbb{Q}(\sqrt{d})$ for an arbitrary squarefree integer $d > 2$. Specialization of the main result will yield the congruences of Ankeny, Artin, and Chowla as well as a stronger version of a theorem of Mordell.

ZHENCHAO GE, University of Waterloo
Irregularities of Dirichlet L-functions and a parity bias in gaps of zeros

The integral of Hardy's Z-function from 0 to T measures the occurrence of its sign changes. Hardy proved that this integral is $o(T)$ from which he deduced that the Riemann zeta-function has infinitely many zeros on the critical line. A. Ivić conjectured this integral is $O(T^{1/4})$ and $\Omega_{\pm}(T^{1/4})$ as $T \to \infty$. These estimates were proved, independently, by M. A. Korolev and M. Jutila.

In this talk, we will show that the analogous conjecture is false for the Z-functions of certain "special" Dirichlet L-functions. In particular, we show that the integral of the Z-function of a Dirichlet L-functions from 0 to T is asymptotic to $c_\chi T^{3/4}$ and we classify precisely when the constant c_χ is nonzero. Experimentally, we find that the L-functions in this (thin) family have a significant and previously undetected bias in the distribution of gaps between their zeros. These phenomena appear to have an arithmetic explanation that corresponds to the non-vanishing of a certain Gauss- type sum.

This is joint work with Jonathan Bober and Micah Milinovich.

FATEMEH JALAVAND, University of Calgary
Geometry of log-unit lattices

The log-unit lattice of a number field is the image of the units of the ring of integers under Minkowski embedding in $\mathbb{R}^n$. Computing the log unit lattice (or a fundamental unit) of a number field is a hard problem and is linked to the problem of computing class numbers which is one of the main tasks of computational algebraic number theory. Knowing the geometry of these lattices may help us to find better ways to compute them.

In this talk, we will discuss the geometry of these lattices. Among different properties, orthogonality and well-roundedness of these lattices are two properties that are more interesting to us. As an example, we will discuss the geometry and shortest vectors of log-unit lattices of totally real biquadratic fields. This is an ongoing project with Jose Cruz.

GREG KNAPP, University of Calgary
Exponential Relations Among Algebraic Integer Conjugates

Products of the form $\alpha_1^{c_1} \cdots \alpha_n^{c_n}$ where the α_i are algebraic are of interest across much of number theory, especially since Baker's results on linear forms in logarithms are widely applicable. In this talk, we explore the scenario where $\alpha_1, \ldots, \alpha_n$ consist only of algebraic integer conjugates, though the α_i need not comprise a full set of algebraic integer conjugates. In particular, for some integers $d \geq 2$ and $1 \leq k \leq d-1$ we describe the set $E_{k,d}$ of all tuples $(c_2, \ldots, c_{k+1}) \in (\mathbb{R}_{\geq 0})^k$ for which $|\alpha_1||\alpha_2|^{c_2} \cdots |\alpha_{k+1}|^{c_{k+1}} \geq 1$ for every tuple of degree d algebraic integer conjugates $\alpha_1, \ldots, \alpha_d$ which are written in descending order of absolute value. Furthermore, for any fixed tuple $(c_2, \ldots, c_{k+1}) \in E_{k,d}$, we ask whether or not there exists a tuple of degree d algebraic integer conjugates $\alpha_1, \ldots, \alpha_d$ (written in descending order of absolute value) so that $|\alpha_1||\alpha_2|^{c_2} \cdots |\alpha_{k+1}|^{c_{k+1}} = 1$. If there does not exist such a tuple, we ask if we can find lower bounds on the quantity $|\alpha_1||\alpha_2|^{c_2} \cdots |\alpha_{k+1}|^{c_{k+1}} - 1$. This talk features joint work with Seda Albayrak, Samprit Ghosh, and Khoa Nguyen.

ENRIQUE NUÑEZ LON-WO, University of Toronto
On the Density of Quadratic Fields with Group of Units in Non-Maximal Orders

For a quadratic number field $K = \mathbf{Q}(\sqrt{d})$ we explore how often $\mathcal{O}_K$ has its group of units in a sub-order $\mathcal{O}$. In particular, when $d \equiv 1 \pmod 4$, we find a lower bound on the lower density of the square-free d such that $\mathbf{Z}[\frac{1+\sqrt{d}}{2}]^\times \neq \mathbf{Z}[\sqrt{d}]^\times$.

PAUL PÉRINGUEY, University of British Columbia
Refinements of Artin's primitive root conjecture

Let $\mathrm{ord_p(a)}$ be the order of a in $(\mathbb{Z}/p\mathbb{Z})^*$. In 1927 Artin conjectured that the set of primes p for which an integer $a \neq -1, \square$ is a primitive root (i.e. $\mathrm{ord_p(a) = p-1}$) has a positive asymptotic density among all primes. In 1967 Hooley proved this conjecture assuming the Generalized Riemann Hypothesis.

In this talk we will study the behaviour of $\mathrm{ord_p(a)}$ as p varies over primes, in particular we will show, under GRH, that the set of primes p for which $\mathrm{ord_p(a)}$ is "k prime factors away" from $p-1$ has a positive asymptotic density among all primes except for particular values of a and k. We will interpret being "k prime factors away" in three different ways, namely $k = \omega(\frac{p-1}{\mathrm{ord_p(a)}})$, $k = \Omega(\frac{p-1}{\mathrm{ord_p(a)}})$ and $k = \omega(p-1) - \omega(\mathrm{ord_p(a)})$, and present conditional results analogous to Hooley's in all three cases and for all integer k.

This is joint work with Leo Goldmakher and Greg Martin.

SHUYANG SHEN, University of Toronto
Enumerative Galois Theory for Trinomials

Much work has been dedicated to studying the frequency of polynomials with given Galois groups. In this talk, we will discuss our approach to this problem for trinomials in particular, and prove bounds for the density of trinomials with certain Galois groups.

ALEXANDER SLAMEN, University of Toronto
A Twisted Variant of Malle's Conjecture

This talk is based on joint work with Brandon Alberts, Helen Grundman, Shilpi Mandal, and Amanda Tucker. Malle's conjecture predicts an asymptotic growth rate for the count of number fields (with a particular Galois group) ordered by discriminant. In the twisted variant, we further stratify the count by demanding that certain fields arise as fixed subfields. This is "twisted" because such extensions are parametrized by particular Galois cohomologies with twisted coefficients. In this talk, I will explore Galois cohomological and embedding-theoretic approaches to the twisted form of Malle's conjecture. We focus on the case of D_8-fields with particular quadratic field fixed by D_4.

YUXUAN SUN, University of Toronto
Approximation Constants and Curves of Best Approximation of Points on Weighted Projective Surfaces

Traditionally, the study of Diophantine approximation involves measuring how well a real number may be approximated by rational numbers using a quantity called the approximation exponent. Over the nineteenth and early twentieth centuries, the approximation exponent of irrational algebraic numbers was refined by many mathematicians, and Klaus Roth (1955) determined that the approximation exponent is 2 for all irrational algebraic numbers. In 2015, David McKinnon and Mike Roth introduced approximation constants for points on algebraic varieties, thereby generalizing the idea of Diophantine approximation via approximation exponents to arbitrary varieties. Further, the approximation constant of a point may be associated to its curves of best approximation, which is a program proposed by McKinnon in 2007.

In this talk, I will present results from a joint project with David McKinnon, Rindra Razafy and Matthew Satriano computing lower bounds of approximation constants of points on a class of weighted projective surfaces. Our technique was based on estimating a related geometric invariant called the effective threshold. I will explain how our lower bounds give useful information about approximation constants of points on the respective surfaces, as well as how one may use these bounds to construct good estimates of curves of best approximation to the respective points. Finally, if I have time, I will present an example where our construction gives a curve of best approximation, as well as an example where the curve of best approximation does not match our construction.

NAIK SUNIL, Queen's University
On some problems in Matsuda monoids

Let F be a field and M be a commutative, torsion-free, cancellative monoid. Let $F[X;M]$ denote the ring of all polynomials with coefficients in F and exponents in M. We say that M is a Matsuda monoid if for every indivisible element α in M, the polynomial $X^\alpha - 1$ is irreducible in $F[X;M]$ for any field F. In this talk, we will discuss recent work on Matsuda monoids that leads to questions in analytic number theory.

WILLIAM VERREAULT, University of Toronto
Moments of random multiplicative functions over functions fields

Little is known about the distribution of the partial sums of random multiplicative functions defined over integers, but the order of magnitude of all moments has been recently determined by Harper. Building on recent work extending multiplicative and probabilistic number theory to the function field setting, we study the even natural moments of partial sums of Steinhaus and Rademacher random multiplicative functions defined over function fields. Using analytic arguments that parallel previous work over the integers as well as new combinatorial arguments special to the function field setting, we obtain an exact expression for the fourth moment and an asymptotic expression for the higher natural moments in the limit as $qN \to \infty$.