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Exponential Relations Among Algebraic Integer Conjugates

Products of the form $\alpha_1^{c_1} \cdots \alpha_n^{c_n}$ where the α_i are algebraic are of interest across much of number theory, especially since Baker's results on linear forms in logarithms are widely applicable. In this talk, we explore the scenario where $\alpha_1, \ldots, \alpha_n$ consist only of algebraic integer conjugates, though the α_i need not comprise a full set of algebraic integer conjugates. In particular, for some integers $d \geq 2$ and $1 \leq k \leq d-1$ we describe the set $E_{k,d}$ of all tuples $(c_2, \ldots, c_{k+1}) \in (\mathbb{R}_{\geq 0})^k$ for which $|\alpha_1||\alpha_2|^{c_2} \cdots |\alpha_{k+1}|^{c_{k+1}} \geq 1$ for every tuple of degree d algebraic integer conjugates $\alpha_1, \ldots, \alpha_d$ which are written in descending order of absolute value. Furthermore, for any fixed tuple $(c_2, \ldots, c_{k+1}) \in E_{k,d}$, we ask whether or not there exists a tuple of degree d algebraic integer conjugates $\alpha_1, \ldots, \alpha_d$ (written in descending order of absolute value) so that $|\alpha_1||\alpha_2|^{c_2} \cdots |\alpha_{k+1}|^{c_{k+1}} = 1$. If there does not exist such a tuple, we ask if we can find lower bounds on the quantity $|\alpha_1||\alpha_2|^{c_2} \cdots |\alpha_{k+1}|^{c_{k+1}} - 1$. This talk features joint work with Seda Albayrak, Samprit Ghosh, and Khoa Nguyen.