Moduli Spaces in Complex and Algebraic Geometry: Recent Developments
Espaces de modules en géométrie complexe et algébrique : Développements récents
(Org: **Robert Cornea** and/et **Ruxandra Moraru** (University of Waterloo))

THOMAS BAIRD, Memorial University
Anti-symplectic involutions of the Hilbert scheme of points on a symplectic surface

Let S be a smooth quasi-projective complex surface. The Hilbert scheme of n points in S, denoted $S^{[n]}$, is a smooth $2n$-dimensional variety which contains the variety of n distinct unordered points as a dense open subvariety.

If S is a symplectic, then $S^{[n]}$ is naturally symplectic. Given an anti-symplectic involution of S, there is an induced involution on $S^{[n]}$ whose fixed point locus is a smooth Lagrangian submanifold. In this talk I explain how to calculate its cohomology and mixed Hodge structure. For the special case $S = \mathbb{C}^2$, this is done using a Morse theory argument borrowed from Ellingsrud-Stromme. For the general case, we adapt an approach due to Gottsche-Soergel involving perverse sheaves.

KUNTAL BANERJEE, University of Saskatchewan
A generalized spectral correspondence

We explore a strong categorical correspondence between isomorphism classes of sheaves of arbitrary rank on a given algebraic curve and twisted pairs on another algebraic curve. We aim to generalize the language of classical spectral correspondence by the annihilating polynomials of pairs. In a particular application, we realize a generic elliptic curve as a spectral cover of the complex projective line and then construct examples of cyclic twisted pairs and co-Higgs bundles on the same curve. Afterwards, by appealing to a composite push-pull projection formula, we explore an iterated version of spectral correspondence for a particular class of spectral covers of the complex projective line through Galois-theoretic arguments. Our explanation relies upon a classification of Galois groups into primitive and imprimitive types. In this context, we revisit a classical theorem of Ritt. This is a joint work with Steven Rayan.

FRANCIS BISCHOFF, University of Regina
The derived moduli stack of logarithmic flat connections

I will present an explicit finite-dimensional model for the derived moduli stack of flat connections on $\mathbb{C}^k$ with logarithmic singularities along a weighted homogeneous Saito free divisor. I will focus in particular on the example of plane curve singularities of the form $x^p = y^q$ and I will discuss the relationship of these moduli spaces with the character varieties of hypersurface complements.

ERIC BOULTER, University of Waterloo
Co-Higgs bundles on Hopf surfaces

Co-Higgs bundles are a variant of Higgs bundles where the twisting bundle is chosen to be the holomorphic tangent bundle instead of the cotangent bundle. We will look at the classification of co-Higgs bundles on a particular family of complex surfaces and discuss how co-Higgs bundles on these surfaces lead to examples of compact holomorphic Poisson 3-folds. Based on joint work with Ruxandra Moraru.

BENOIT CHARBONNEAU, University of Waterloo
Deformed Hermitian-Yang-Mills on full flags

With Gonçalo Oliveira and Rosa Sena-Dias, we study the deformed Hermitian-Yang-Mills equation on the full flag manifold, both in rank one and in higher rank.

EMILY CLIFF, Université de Sherbrooke
Moduli spaces of principal 2-group bundles and a categorification of the Freed–Quinn line bundle

A 2-group is a categorified version of a group: a category with a multiplication operator, for which all group axioms hold up to natural isomorphism. Similarly, there is a notion of principal bundle for a 2-group. We define the moduli space of principal 2-group bundles, and prove that it is gives a 2-fibration over the moduli space of principal bundles for an ordinary group G. Moreover, when G is finite, this 2-fibration provides a categorification of the Freed–Quinn line bundle, a mapping class group equivariant line bundle arising in Dijkgraaf–Witten theory for the finite group G. This is joint work with Daniel Berwick-Evans, Laura Murray, Apurva Nakade, and Emma Phillips.

LISA JEFFREY, University of Toronto
Character Varieties

Character varieties can be regarded in terms of flat connections on oriented 2-manifolds, or in terms of representations of the fundamental group of a 2-manifold into a Lie group. They have a Poisson structure. The Poisson structure was originally defined by Bill Goldman (1984) or Atiyah-Bott (1983).

I will outline the origin of the Poisson structure. I will also describe how to define an almost complex structure on the symplectic leaves, in some situations.

Some parts of the material presented are joint with Indranil Biswas, Jacques Hurtubise and Sean Lawton. Other parts are joint with Yukai Zhang.

ELANA KALASHNIKOV, University of Waterloo
Degenerations of Kronecker moduli spaces

Kronecker moduli spaces are simple, and yet much more complicated, generalizations of type A Grassmannians. In this talk, I'll explain recent joint work with Liana Heuberger on the structure of the Cox ring of Kronecker moduli spaces, and applications to degenerations and mirror symmetry.

MOHSEN KARKHEIRAN, University of Alberta
Heterotic-II duality from mirror symmetry.

The four dimensional duality between type IIA and Heterotic string theories is well known for decades. This duality involves the degeneration of Calabi-Yau manifolds which are similar to the DHT conjecture in mirror symmetry. In this work we show this duality is indeed closely related to mirror symmetry, and we investigate the consequences of that.

DEREK KREPSKI, University of Manitoba
Lie 2-algebras of infinitesimal symmetries of bundle gerbes

In this talk, we discuss how the infinitesimal symmetries of a bundle gerbe over a smooth manifold M naturally form a Lie 2-algebra. These symmetries are closely related to Lie 2-algebras naturally associated to a closed 3-form $\chi \in \Omega^3(M)$: the Poisson Lie 2-algebra of observables on a 2-plectic manifold (M, χ), the Lie 2-algebra of sections of the exact Courant algebroid $TM \oplus T^*M$ with χ-twisted Courant bracket, and the so-called Atiyah Lie 2-algebra associated to the Lie algebra action of vector fields on smooth functions. This is joint work with Jennifer Vaughan and Dinamo Djounvouna.

HAGGAI LIU, Simon Fraser University
Moduli Spaces of Weighted Stable Curves and their Fundamental Groups

The Deligne-Mumford compactification, $\overline{M_{0,n}}$, of the moduli space of n distinct ordered points on $\mathbb{P}^1$, has many well understood geometric and topological properties. For example, it is a smooth projective variety over its base field. Many interesting

properties are known for the manifold $\overline{M_{0,n}}(\mathbb{R})$ of real points of this variety. In particular, its fundamental group, $\pi_1(\overline{M_{0,n}}(\mathbb{R}))$, is related, via a short exact sequence, to another group known as the cactus group. Henriques and Kamnitzer gave an elegant combinatorial presentation of this cactus group.

In 2003, Hassett constructed a weighted variant of $\overline{M_{0,n}}(\mathbb{R})$: For each of the n labels, we assign a weight between 0 and 1; points can coincide if the sum of their weights does not exceed one. We seek combinatorial presentations for the fundamental groups of Hassett spaces with certain restrictions on the weights. In particular, we express the Hassett space as a blow-down of $\overline{M_{0,n}}$ and modify the cactus group to produce an analogous short exact sequence. The relations of this modified cactus group involves extensions to the braid relations in S_n. To establish the sufficiency of such relations, we consider a certain cell decomposition of these Hassett spaces, which are indexed by ordered planar trees.

CHRISTOPHER MAHADEO, University of Illinois at Chicago

Topological recursion and twisted Higgs bundles

Prior works relating meromorphic Higgs bundles to topological recursion have considered non-singular models that allow the recursion to be carried out on a smooth Riemann surface. I will discuss some recent work where we define a "twisted topological recursion" on the spectral curve of a twisted Higgs bundle, and show that the g=0 components of the twisted recursion relate to the Taylor expansion of the period matrix of the spectral curve, mirroring a result of for ordinary Higgs bundles and topological recursion.

STEVE RAYAN, University of Saskatchewan

Resolutions of finite quotient singularities and quiver varieties

Finite quotient singularities have a long history in mathematics, intertwining algebraic geometry, hyperkähler geometry, representation theory, and integrable systems. I will highlight the correspondences at play here and how they culminate in Nakajima quiver varieties, a class of moduli spaces that provide a useful testing ground for ideas in geometric representation theory and physics. I will motivate some recent work of G. Bellamy, A. Craw, T. Schedler, H. Weiss, and myself in which we show that, remarkably, all of the resolutions of a particular finite quotient singularity are realized as a certain Nakajima quiver variety, namely that of the 5-pointed star-shaped quiver. I will place this work in the wider context of the search for McKay-type correspondences for finite subgroups of SL(n,$\mathbb{C}$) on the one hand, and of the construction of finite-dimensional-quotient approximations to meromorphic Hitchin systems and their integrable systems on the other hand. The Hitchin system perspective draws upon my prior joint works with each of J. Fisher and L. Schaposnik, respectively. Time permitting, I will speculate upon the symplectic duality of Higgs and Coulomb branches in this setting.