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Effect of a variable bottom topography on surface water waves

We investigate the effect of the bottom topography on the evolution of surface waves. It is a problem of significance for ocean dynamics in coastal regions where waves are strongly affected by the topography. The literature on models of free surface water waves over a variable depth is extensive. In the presence of topography, there are several asymptotic scaling regimes of interest, including long-wave hypotheses for the evolution of the free surface, and short scale and/or long scale variations in the variable bottom. A central object in the analysis of the water wave problem is the Dirichlet-Neumann operator and our study concerns its spectrum in the context of the water wave system linearized near equilibrium in a domain with a variable bottom assumed to be a smooth periodic function. We use the analyticity of the Dirichlet-Neumann operator with respect to the bottom variation and combine it with general properties of elliptic systems and spectral theory for self-adjoint operators to develop a Bloch-Floquet theory and describe the structure of its spectrum. We find that, under some conditions on the bottom variations, the spectrum is composed of bands separated by gaps which are zones of forbidden energies, and we give explicit formulas for their sizes and locations.