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An Application of Finite Blaschke Products in Numerical Range Studies

Let T be an operator on a Hilbert space H with numerical radius $w(T) \leq 1$. According to a theorem of Berger and Stampfli, if f is a function in the disk algebra such that $f(0) = 0$, then $w(f(T)) \leq \|f\|_\infty$. We give a new and elementary proof of this result using finite Blaschke products. A well-known result relating numerical radius and norm says $\|T\| \leq 2w(T)$. We obtain a local improvement of this estimate, namely,

$$\|Tx\|^2 \leq 2 + 2\sqrt{1 - |\langle Tx, x\rangle|^2}, \qquad (x \in H, \ \|x\| \leq 1),$$

Whenever $w(T) \leq 1$. Using this refinement, we give a simplified proof of Drury's teardrop theorem, which extends the Berger-Stampfli theorem to the case $f(0) \neq 0$.

Joint work with T. Ransford and H. Klaja