CECILE PIRET, Michigan Technological University

Computing generalized hypergeometric functions in the complex plane using an end-corrected trapezoidal rule

Generalized hypergeometric functions ${}_pF_q$ are ubiquitous in the scientific and engineering fields, for which their accurate evaluation is essential. Although a myriad of algorithms exists for their evaluation in the complex plane, most commonly with low parameters p and q, no general framework exists that is adaptable to low and high p and q values, wide ranges of coefficients, and small to large evaluation domains. This results in an intricate patchwork of algorithms with sometimes radically different orders of accuracy and computational cost. We introduce a high order method ($> 20^{th}$ order) which addresses this issue by its wide ranging validity. It is based on the Euler's integral transformation of the hypergeometric functions formula, and therefore shares its parameters ($p \leq q+1$) and coefficient constraints. The method is based on an end-corrected trapezoidal rule applied to singular integrals, first introduced in [1] in the context of fractional derivatives.

[1] B. Fornberg and C. Piret, Computation of Fractional Derivatives of Analytic Functions. J Sci Comput 96, 79 (2023).